**Mathematics 122 Final** Name: _____

**You are to use your own calculator, no sharing.**

**Show your work to get credit.** This means that if you use your calculator to solve a problem, then you have to write a sentence telling how you used it to do the calculations. (That is if you graphed it and zoomed in then say that is what you did etc.)

1. (15 Points) Find the following antiderivatives.
   (a) $\int (3x^4 - 2x^3 + x^2 - 4x + 5)\, dx$ _____

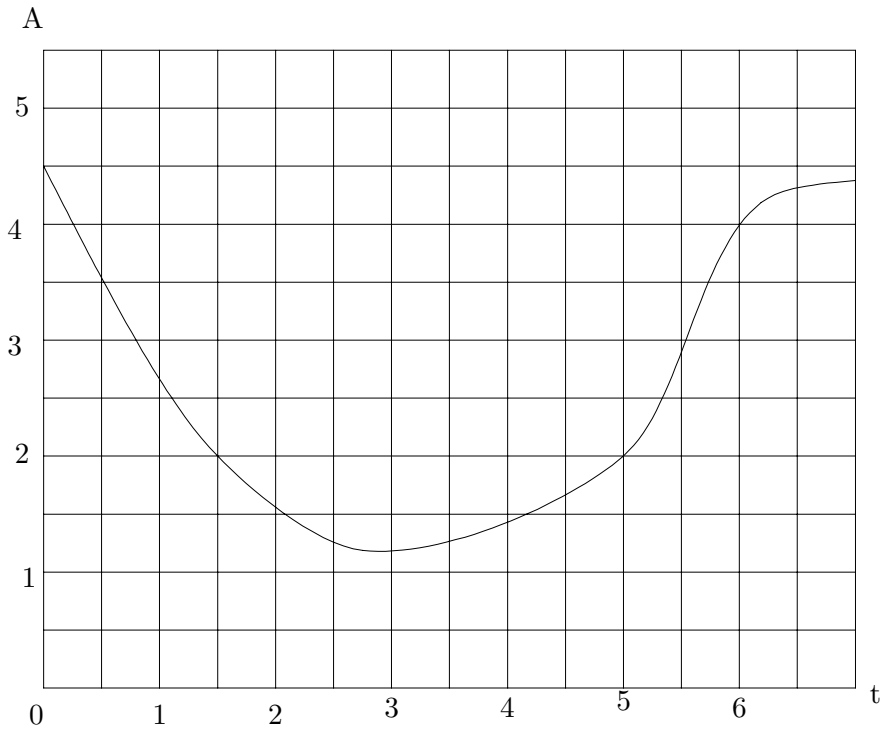
   (b) $\int \left(3\sqrt{t} + \frac{4}{t^3}\right) dt$ _____

   (c) $\int e^{3y}\, dy$ _____

2. (15 Points) Let $y = 4x^3 - 3x^2 - 6x + 3$
   (a) Find the critical points of $y$ and classify as to local maximums or minimums.

   (b) Find the inflection points of $y$.

3. (10 Points) The following is a graph of $A$ as function of $t$.




(a) What is the value of $A$ when $t = 3$?

(b) For what values of $t$ is $A = 2$?

(c) What is the average rate of change of $A$ with respect to $t$ between $t = 1.5$ and $t = 6$?

(d) What it the value of the derivative $\dfrac{dA}{dt}$ when $t = 1.5$?

(e) What is the largest that $A$ becomes?

(f) What value of $t$ makes $A$ the largest?

4. (10 Points) Let $f(x) = \dfrac{3 + 2^x}{x^3 + 2}$.

(a) Compute the derivative $f'(3)$. (You may use your calculator.)

(b) What is the equation of the tangent line to $y = f(x)$ at the point where $x = 3$?

5. (10 Points) Corresponding values of $p$ and $q$ are given by the table:

| $p$ | 7 | 10 | 13 | 16 |
|---|---|---|---|---|
| $q$ | 3 | 5 | 7 | 9 |

Assuming that the relationship between $p$ and $q$ is linear answer the following:

(a) Find $p$ as a linear function of $q$ _____

(b) Find the value of $q$ when $p = 20$ $q=$ _____

6. (10 Points) The following table gives the some of the values of an exponential function $P = P(t)$.

| $t$ | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| $P(t)$ | 5.000 | 9.250 | 17.113 | 31.658 | 58.568 | 108.350 |

(a) What is the factor by which $P(t)$ changes when $t$ is increased by 1?

_____

(b) What is a formula for $P(t)$ as a function of $t$?

$P(t) =$ _____

(c) What is the value of $P(t)$ when $t = 10$?

$P(10) =$ _____

7. (10 Points) Sketch the graph of a function $y = f(x)$ so that
   - $f(2) = 2$,
   - $f(5) = 6$,
   - $f(7) = -1$,
   - $f'(x) < 0$ for $x < 2$,
   - $f'(x) > 0$ for $2 < x < 5$,
   - $f'(x) < 0$ for $5 < x < 7$, and
   - $f'(x) > 0$ for $7 < x$.

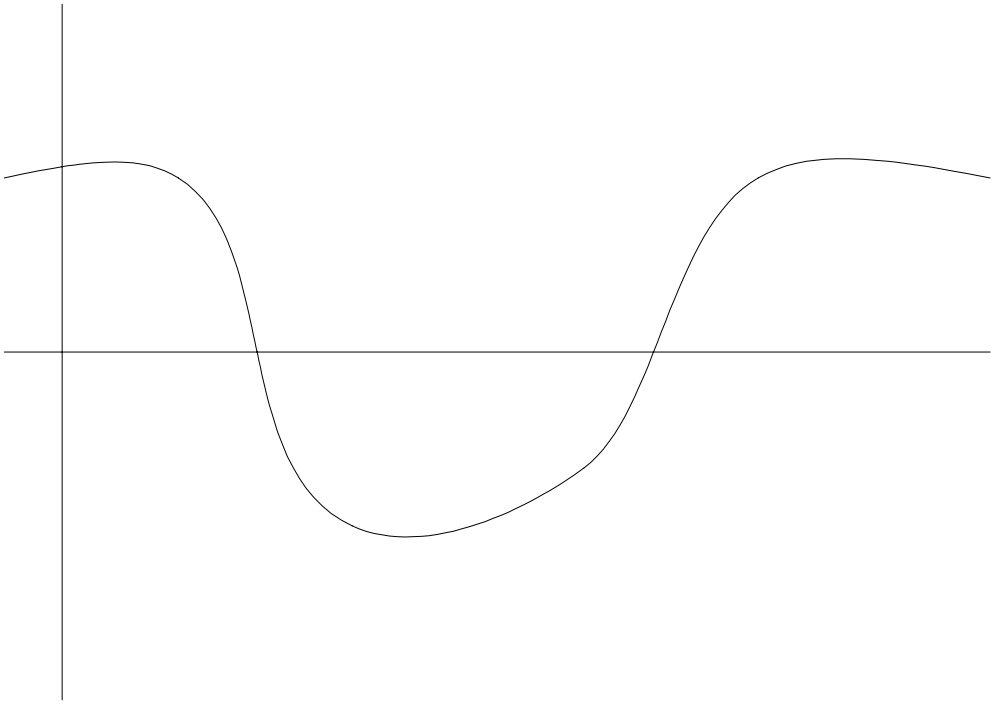
   Your graph should not have any sharp corners.
8. (10 Points) Draw Graphs of functions that satisfy the following:
   (a) Is increasing at an increasing rate.

   (b) Is decreasing and concave down.

   (c) Has $f'(x) > 0$ and $f''(x) < 0$

   (d) has $f''(x) > 0$, $f(1) = 2$ and $f'(1) = 0$.

9. (10 Points) For the function given by the following graph sketch a graph of the derivative on the same set of axis.



10. (10 Points) Let a function $w(t)$ be given by the table

| $t$ | 2 | 4 | 6 | 8 |
|---|---|---|---|---|
| $w(t)$ | 11.3 | 10.62 | 8.71 | 5.38 |

(a) Approximate the derivative at the points 3, 5, 7.

| $t$ | 3 | 5 | 7 |
|---|---|---|---|
| $w'(t)$ | | | |

(b) Is the second derivative $w''(t)$ positive or negative? Explain your answer.

(c) Estimate $w''(4)$ $\qquad w''(4) \approx$ _____

11. (15 Points) A car goes 30mph for 45min. It then goes for 50mph for 30min and slows down to 40mph for an hour.
    (a) Sketch a graph of the speed of the care as a function of time

    (b) How far did the car travel in the first hour of the trip? _____

    (c) What was the average speed of the car during the entire trip. _____

    (d) Sketch a graph of distance traveled as a function of time.

12. (15 Points) Compute the derivatives of the following functions (no simplification required):

(a) $y = 5x^4 - 9x^3 + 3x^2 + 2x - 7$ $\qquad y' =$ _____

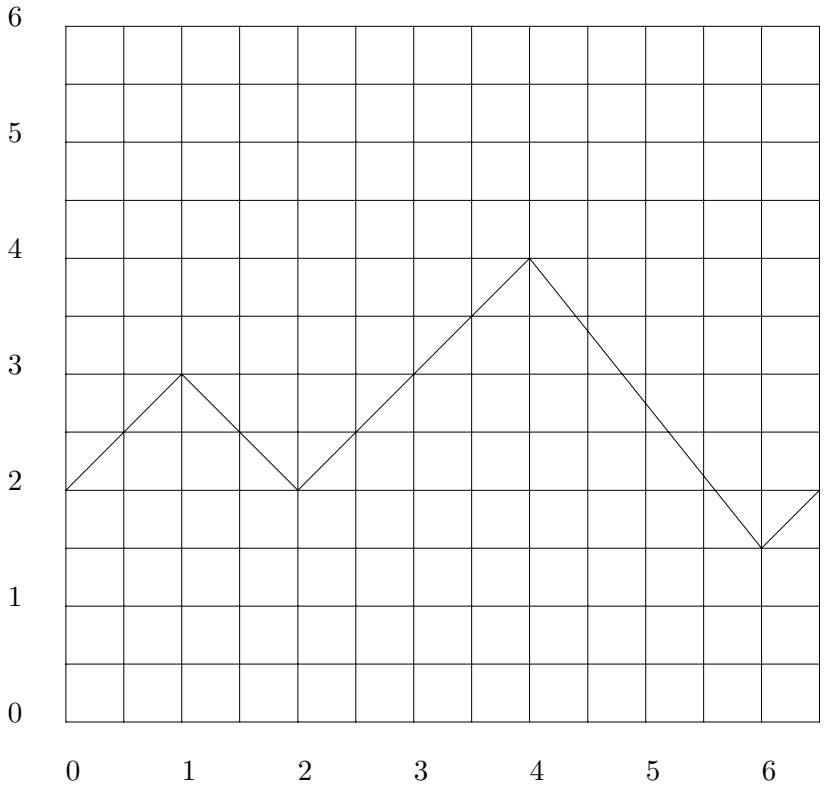
(b) $R(t) = 3\sqrt{t} + 2e^{4t}$ $\qquad R'(t) =$ _____

(c) $H(t) = 2t^3 e^t$ $\qquad H'(t) =$ _____

(d) $P(r) = 3(r^5 + 2r)^4$ $\qquad P'(r) =$ _____

(e) $I(x) = 2\ln(x^2 + x)$ $\qquad I'(x) =$ _____

13. (10 Points) Below is the graph of $f(t) = F'(t)$ (that is the graph is if the rate of change of $F$).




(a) If $F(0) = 2.5$ then what is $F(4)$? $\qquad F(5) =$ _____

(b) How much does $F$ change between $t = 1$ and $t = 5$? _____

*Have a nice summer!*