**Mathematics 122 Final Exam.** Name: _____

**You are to use your own calculator, no sharing.**

**Show your work to get credit.** This means that if you use your calculator to solve a problem, then you have to write a sentence telling how you used it to do the calculations. (That is if you graphed it and zoomed in then say that is what you did etc.

1. (25 points) Find the following derivatives:

   (a) $y = 5x^4 - 9x^3 + 3x^2 + 2x - 7$            $y' =$ _____

   (b) $R(t) = 4\sqrt{t} + 6e^{2t}$            $R'(t) =$ _____

   (c) $H(t) = 3t^2 e^{-t}$            $H'(t) =$ _____

   (d) $P(r) = 3(r^3 - 7r)^4$            $P'(r) =$ _____

   (e) $I(x) = 2\ln(x^2 + x)$            $I'(x) =$ _____

2. (5 points) Find the equation of the line through $(1, -2)$ with slope 4.

_____

3. (10 points) Find the equation of the tangent line to $y = x^2 + 2x - 5$ at the point where $x = 1$

_____

4. (20 points) In 1860 a ship lands on a small South sea island and 100 rats jump ship and start a population of rats on the island. By 1870 there are 350 rats on the island. Assume that the population of rats is a linear function of time.

(a) What is the average growth of the population of rats? (Be sure to include units on your answer.)

_____

(b) Give a formula for the number of rats $t$ years after the 1860.

_____

(c) Predict how many there will be on the island in 1900.

_____

(d) Make a prediction of what year there will be 1000 rats on the island.

_____

5. (10 Points) Let $y = f(x)$ have the following graph.
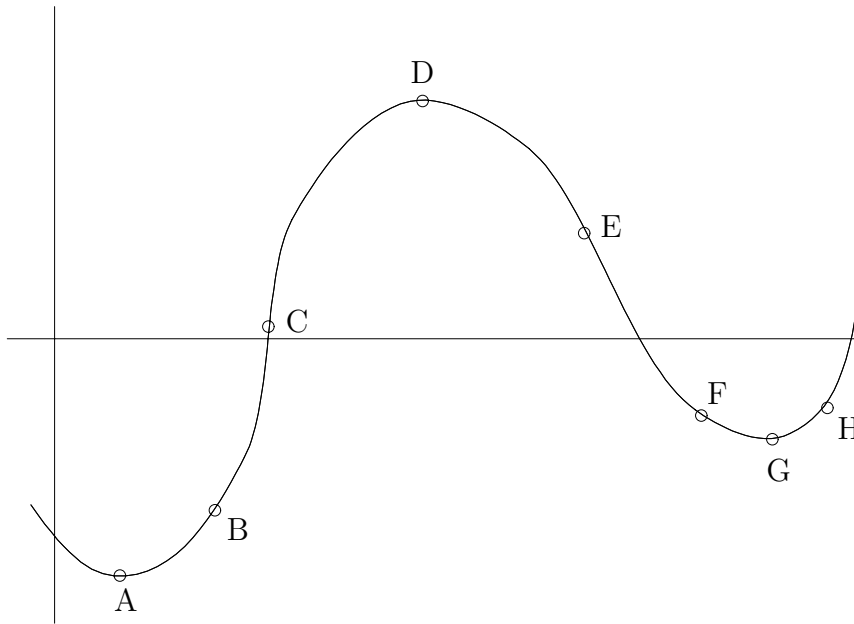


(a) At which of the labeled points is $f'(x) > 0$? _____

(b) At which is the labeled points is $f'(x) < 0$? _____
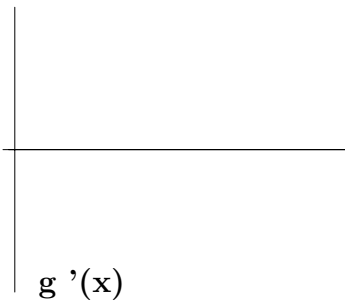
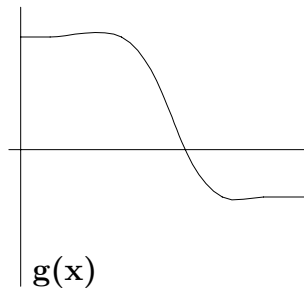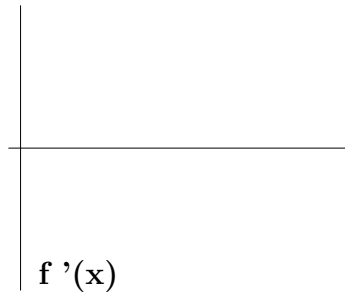(c) At which is the labeled points is $f'(x) = 0$? _____
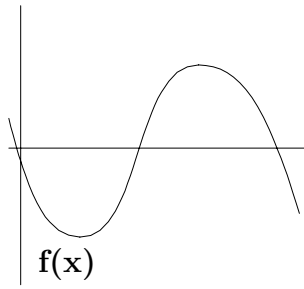
(d) At which of the labeled points if $f(x)$ largest? _____

(e) At which of the labeled points is $f'(x)$ largest? _____

6. (5 points) Let $f(t) = (2^t)/(4 + t^3)$. Then estimate $f'(2)$ You may use your calculator.

$$f'(2) \approx \text{_____}$$

7. (10 Points) Below are the graphs of $y = f(x)$ and $y = g(x)$. Sketch the graphs of $y = f'(x)$ and $y = g'(x)$.

f(x)

f '(x)

g(x)

g '(x)

8. (10 Points) Draw the graph of a function $y = f(x)$ so that $f'(x) < 0$ for $x < 3$, $f'(x) > 0$ for $3 < x < 7$ and $f'(x) < 0$ for $7 < x$, $f(3) = -1$, and $f(7) = 4$.

9. (20 Points) Draw Graphs of functions that satisfy the following:
   (a) Has $f'(x) > 0$ and $f''(x) < 0$

   (b) has $f''(x) < 0$ and $f'(1) = 0$.

   (c) Is increasing at an increasing rate.

   (d) Is decreasing and concave up.

10. (15 Points) Let the function $u = f(t)$ have its values as in the following table:

| $t$ | .5 | 1.0 | 1.5 | 2.0 | 2.5 |
|-----|------|-----|------|-----|------|
| $f(t)$ | 1.75 | 1.0 | 0.75 | 1.0 | 1.75 |

   (a) What is that average rate of change of $f$ on the interval from $t = 1.0$ to $t = 2.5$?

   (b) What is an estimate for $f'(2.5)$?

   (c) What is a good estimate for $f(3)$?

11. (20 points) Compute the following (you may use your calculator).

(a) $\int_1^4 3(1.7)^t \, dt$ _____

(b) $\int_{-2}^2 (x^3 - 2x^2) \, dx$ _____

(c) $\int_0^3 \frac{1 - 2^x}{1 + x^2} \, dx$ _____

(d) The average value of $f(x) = x(3 - x)$ on the interval $[0, 3]$.

_____

12. (5 points) A function $f(t)$ has values given by the table:

| $t$    | 1.00 | 1.25 | 1.5 | 1.75 | 2.00 |
|--------|------|------|-----|------|------|
| $f(t)$ | 3.2  | 3.5  | 3.7 | 3.9  | 4.0  |

Estimate $\int_1^2 f(t) \, dt$.           $\int_1^2 f(t) \, dt \approx$ _____

13. (15 points) Let $H(t)$ be the cost per day. measured dollars per day, to heat a house, starting with $t = 0$ on January 1, 1998. Assume that $H(t)$ is given by

$$H(t) = \begin{cases} 2.0, & 0 \le t < 30 \\ 1.5, & 30 \le t \le 75 \\ 1.0 & 75 < t \le 120. \end{cases}$$

(a) What are the units on $\int_0^{120} H(t)\, dt$?

Units are: _____

(b) What is the interpretation of $\int_0^{120} H(t)\, dt$?

(c) What is the value of $\int_0^{120} H(t)\, dt$? (HINT: It might help to graph $y = H(t)$.)

$$\int_0^{120} H(t)\, dt = \underline{\hspace{3cm}}$$

(d) What is the average value of $H(t)$ on the interval $0 \le t \le 90$?

Average = _____

14. (15 points) Compute the following anti-derivatives.

(a) $\int (4x^5 - 3x^2) \, dx$ _____

(b) $\int e^{5t} \, dt$ _____

(c) $\int 3\sqrt{x} \, dx$ _____

15. (15 points) An investment increases at the rate of 10% per year. Assume that originally $1000.00 are invested.

(a) What is the value of the investment after 2 years.

_____

(b) Give a formula for the value of the investment after $t$ years.

_____

(c) How long before the value of the investment doubles?

_____

*Have a Nice Holiday.*