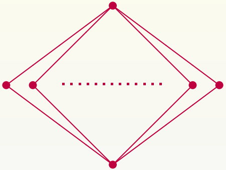
Packing Posets in the Boolean Lattice



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For a poset P, we consider how large a family $\mathcal{F}$ of subsets of $[n] := \{1, \ldots, n\}$ we may have in the Boolean Lattice $\mathcal{B}_n : (2^{[n]}, \subseteq)$ containing no (weak) subposet P. We are interested in determining or estimating $\boxed{\mathrm{La}(n, P) := \max\{|\mathcal{F}| : \mathcal{F} \subseteq 2^{[n]}, P \not\subset \mathcal{F}\}.}$

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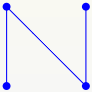
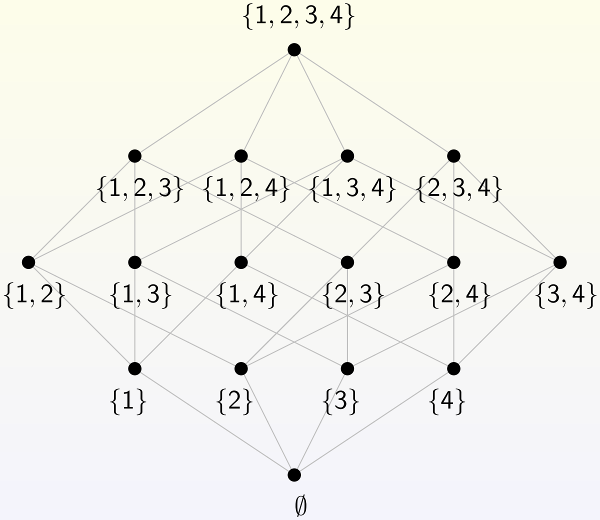
Example

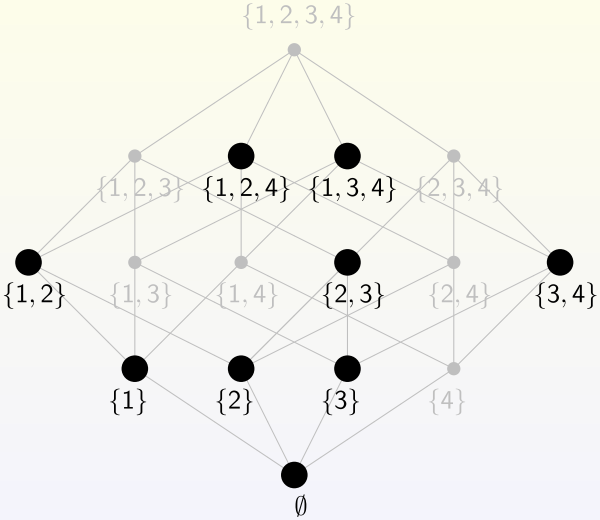


For the poset $P = \mathcal{N}$, $\mathcal{F} \not\supset \mathcal{N}$ means $\mathcal{F}$ contains no 4 subsets A, B, C, D such that $A \subset B$, $C \subset B$, $C \subset D$. Note that $A \subset C$ is allowed: The subposet does not have to be induced.

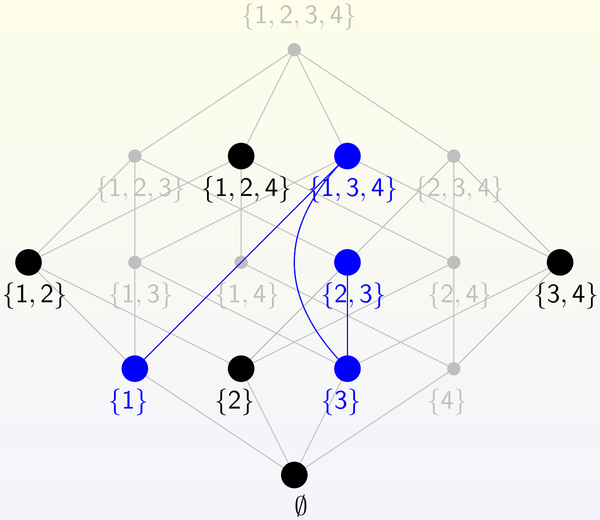
The Boolean Lattice $\mathcal{B}_4$



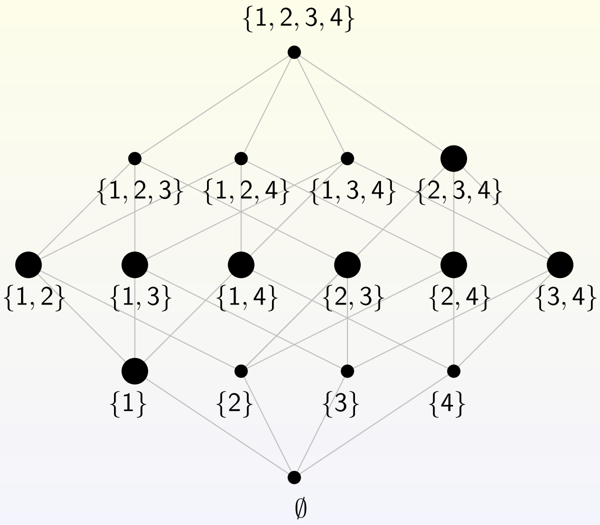
A Family of Subsets $\mathcal{F}$ in $\mathcal{B}_4$



$\mathcal{F}$ contains the poset $\mathcal{N}$



A Large $\mathcal{N}$-free Family in $\mathcal{B}_4$



Given a finite poset P, we are interested in determining or estimating $\boxed{\mathrm{La}(n,P) := \max\{|\mathcal{F}| : \mathcal{F} \subseteq 2^{[n]}, P \not\subset \mathcal{F}\}.}$

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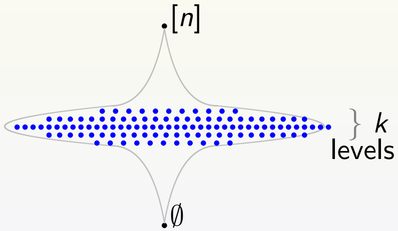
For many posets, $\mathrm{La}(n,P)$ is exactly equal to the sum of middle k binomial coefficients, denoted by $\Sigma(n,k)$.

Moreover, the largest families may be $\mathcal{B}(n,k)$, the families of subsets of middle k sizes.



Foundational results: Let $\mathcal{P}_k$ denote the k-element chain (path poset).

Theorem (Sperner, 1928)

For all n,

$$\mathrm{La}(n, \mathcal{P}_2) = \binom{n}{\lfloor \frac{n}{2} \rfloor},$$

and the extremal families are $\mathcal{B}(n, 1)$.

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Theorem (Erdős, 1945)

For general k and n,

$$\mathrm{La}(n, \mathcal{P}_k) = \Sigma(n, k-1),$$

and the extremal families are $\mathcal{B}(n, k-1)$.

Excluded subposet P | $\mathrm{La}(n, P)$

$\mathcal{P}_2$

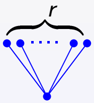
$\binom{n}{\lfloor \frac{n}{2} \rfloor}$ [Sperner, 1928]

Path $\mathcal{P}_k$, $k \geq 2$

$\Sigma(n, k-1)$

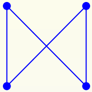
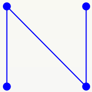
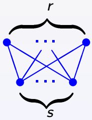
$\sim (k-1)\binom{n}{\lfloor \frac{n}{2} \rfloor}$ [P. Erdős, 1945]

r-fork $\mathcal{V}_r$



$\sim \binom{n}{\lfloor \frac{n}{2} \rfloor}$ [Katona-Tarján, 1981]

[DeBonis-Katona 2007]

Excluded subposet P	$\mathrm{La}(n,P)$	
Butterfly B 	$\Sigma(n,2)$ $\sim 2\binom{n}{\lfloor \frac{n}{2} \rfloor}$	[DeBonis-Katona-Swanepoel, 2005]
$\mathcal{N}$ 	$\sim \binom{n}{\lfloor \frac{n}{2} \rfloor}$	[G.-Katona, 2008]
$\mathcal{K}_{r,s}(r,s \geq 2)$  $r,s \geq 2$	$\sim 2\binom{n}{\lfloor \frac{n}{2} \rfloor}$	[De Bonis-Katona, 2007]

Asymptotic behavior of $\mathrm{La}(n, P)$

Definition

$\pi(P) := \lim_{n\to\infty} \frac{\mathrm{La}(n,P)}{\binom{n}{\lfloor \frac{n}{2} \rfloor}}$.

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Conjecture (G.-Lu, 2008)

For all P, $\pi(P)$ exists and is integer.

When Saks and Winkler (2008) observed what $\pi(P)$ is in known cases, it led to the stronger

Conjecture (G.-Lu, 2009)

For all P, $\pi(P) = e(P)$, where

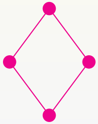
Definition

$e(P) :=$ max m *such that for all n, $P \not\subset \mathcal{B}(n, m)$.*

On the Diamond $\mathcal{D}_2$

Problem

Despite considerable effort it remains open to determine the value $\pi(\mathcal{D}_2)$ or even to show it exists!



The conjectured value of $\pi(\mathcal{D}_2)$ is its lower bound, $e(\mathcal{D}_2) = 2$.

Successive upper bounds on $\pi(\mathcal{D}_2)$:

2.5 [G.-Li, 2007]
2.296 [G.-Li-Lu, 2008]
2.283 [Axenovich-Manske-Martin, 2011]
2.273 [G.-Li-Lu, 2011]
2.25 [Kramer-Martin-Young, 2012]

Three level problem

To make things simpler, what if we restrict attention to D_2-free families in the middle three levels of the Boolean lattice B_n? We should get better upper bounds on $|\mathcal{F}|/\binom{n}{\lfloor \frac{n}{2} \rfloor}$:

2.207 [Axenovich-Manske-Martin, 2011]
2.1547 [Manske-Shen, 2012]
2.1512 [Balogh-Hu-Lidický-Liu, 2012]

Excluding a Family of Posets

Let us now generalize $\mathrm{La}(n, P)$.

We consider $\boxed{\mathrm{La}(n, \{P_i\}) := \max\{|\mathcal{F}| : \mathcal{F} \subseteq 2^{[n]}, \quad \forall i P_i \not\subset \mathcal{F}\}.}$

In words, it is the maximum size of a family $\mathcal{F} \subseteq \mathcal{B}_n$ that contains no copy of any poset $P_i \in \{P_i\}$.

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What is $\mathrm{La}(n, \{\mathcal{V}, \Lambda\})$, where $\mathcal{V} = \mathcal{V}_2$?

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Theorem (Katona, Tarján (1983))

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Although solved, let us think about this question. A family with neither $\mathcal{V}$ nor Λ is constructed from subposets $\mathcal{B}_0$ and $\mathcal{B}_1$ that are not only disjoint, but are unrelated. It means that no element of one is related to an element of another.

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Our question becomes: What is the largest size of a family in $\mathcal{B}_n$ constructed from pairwise unrelated copies of $\mathcal{B}_0$ and $\mathcal{B}_1$?

Maximum Packings

We consider

$\mathrm{Pa}(n, \{P_i\}) := \max\{|\mathcal{F}| : \mathcal{F} \subseteq 2^{[n]}\}$,
where each component of $\mathcal{F}$ is a copy of some poset in the collection $\{P_i\}$. The collection may be infinite.

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This can be viewed as a generalization of the $\mathrm{La}(n, \{Q_j\})$ problem:

$\mathrm{La}(n, \{Q_j\}) = \mathrm{Pa}(n, \{P_i\})$,

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We have that $\mathrm{La}(n, \{\mathcal{V}, \Lambda\}) = \mathrm{Pa}(n, \{\mathcal{B}_0, \mathcal{B}_1\})$.

Old Examples with New Notation

Problem

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Note: Katona independently introduced what is equivalent to $\mathrm{Pa}(n, P)$ in 2010.

Theorem (Sperner (1928))

$\mathrm{Pa}(n, \mathcal{B}_0)$ is $\binom{n}{\lfloor \frac{n}{2} \rfloor}$.

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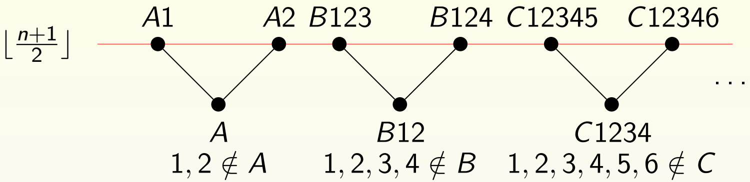
Theorem (Sperner (1928))
$\mathrm{Pa}(n, \mathcal{B}_0)$ *is* $\binom{n}{\lfloor \frac{n}{2} \rfloor}$.

Theorem (G., Stahl, Trotter (1984))
For the chain (or path) $\mathcal{P}_k$ on $k \geq 1$ elements,

$$\mathrm{Pa}(n, \mathcal{P}_k) = k \binom{n-k+1}{\lfloor \frac{n-k+1}{2} \rfloor}.$$

The maximum number of unrelated copies of $\mathcal{P}_k$ in $\mathcal{B}_n$ is asymptotic to $\frac{1}{2^{k-1}} \binom{n}{\lfloor \frac{n}{2} \rfloor}$.

Example: Packing many copies of $\mathcal{V}$



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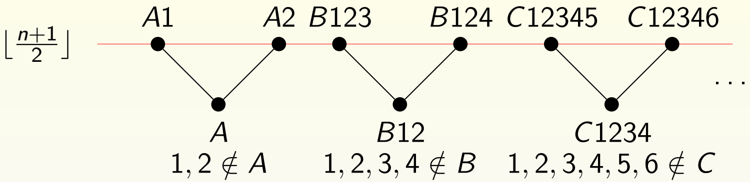

$$\mathrm{Pa}(n, \mathcal{V})/|\mathcal{V}| \geq \left[\binom{n-2}{\frac{n-1}{2}} + \binom{n-4}{\frac{n-1}{2}-2} + \binom{n-6}{\frac{n-1}{2}-4} + \dots \right]$$

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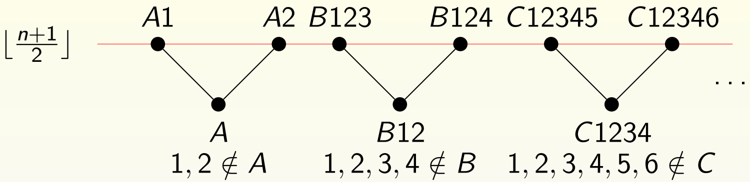

$$\mathrm{Pa}(n, \mathcal{V})/|\mathcal{V}| \geq \left[\binom{n-2}{\frac{n-1}{2}} + \binom{n-4}{\frac{n-1}{2} - 2} + \binom{n-6}{\frac{n-1}{2} - 4} + \dots \right]$$

$$\sim \left[\frac{1}{2^2} \binom{n}{\lfloor \frac{n}{2} \rfloor} + \frac{1}{2^4} \binom{n}{\lfloor \frac{n}{2} \rfloor} + \frac{1}{2^6} \binom{n}{\lfloor \frac{n}{2} \rfloor} + \dots \right] = \frac{1}{3} \binom{n}{\lfloor \frac{n}{2} \rfloor}.$$

Example: Packing many copies of $\mathcal{V}$

On the other hand, by counting how many maximal chains in $\mathcal{B}_n$ hit a $\mathcal{V}$, we get the same expression as an asymptotic upper bound, and deduce

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Discussing this with Richard Anstee, we were led to make the

Conjecture

For every poset P, there is some integer $c(P)$ such that $\mathrm{Pa}(n, P)/|P| \sim \frac{1}{c(P)}\binom{n}{\lfloor \frac{n}{2} \rfloor}$ as $n \to \infty$.

Convex Closure and Main Theorem

For a family $\mathcal{F} \subseteq \mathcal{B}_n$, its *convex closure* is the family

$$\overline{\mathcal{F}} := \{S \in \mathcal{B}_n | A \subseteq S \subseteq B \text{ for some } A, B \in \mathcal{F}\}.$$

Notice that if a family G is unrelated to a family F, then G is also unrelated to $\overline{\mathcal{F}}$.

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Theorem (D., G. (2013), and Katona, Nagy (2013))
As n goes to infinity, $\mathrm{Pa}(n, P) \sim \frac{|P|}{c(P)} \binom{n}{\lfloor \frac{n}{2} \rfloor}$.

Proof Sketch: Upper Bound of $\mathrm{Pa}(n, P)$

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No full (maximum) chain in $\mathcal{B}_n$ meets more than one closure of a copy of P. If each closure of a copy of P meets at least x full

chains, then

$$\frac{\mathrm{Pa}(n, P)}{|P|} x \le n!;$$

$$\mathrm{Pa}(n, P) \le |P| \frac{n!}{x}.$$

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We need to show the closure of a copy of P meets at least $c(P)\lfloor n/2 \rfloor!\lceil n/2 \rceil!$ full chains asymptotically.

Upper Bound of $\mathrm{Pa}(n, P)$

Let $a(n, m)$ denote the largest integer such that any family $\mathcal{F} \subseteq \mathcal{B}_n$, $|\mathcal{F}| = m$, meets at least $a(n, m)$ full chains.

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Proof of Lemma: Inclusion/Exclusion: Let $\mathcal{F} \subseteq \mathcal{B}_n$ be a family of size m that meets $a(n,m)$ full chains. Let $b(\{A_1, \ldots, A_k\})$ be the number of full chains in $\mathcal{B}_n$ that meet all of the sets in $\{A_1, \ldots, A_k\}$.

$$a(n,m) \geq \sum_{A \in \mathcal{F}} b(\{A\}) - \sum_{A_1, A_2 \in \mathcal{F}} b(\{A_1, A_2\}).$$

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$$\geq \sum_{A \in \mathcal{F}} \left(b(\{A\}) - \sum_{\substack{A_1 \in \mathcal{F} \\ \left|\frac{n}{2} - |A|\right| \geq \left|\frac{n}{2} - |A_1|\right|}} b(\{A, A_1\}) \right)$$

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$$\geq \sum_{A \in \mathcal{F}} b(\{A\}) \left(1 - \frac{2m}{n} \right) \sim \sum_{A \in \mathcal{F}} b(\{A\}) \geq m \lfloor (n/2) \rfloor ! \lceil (n/2) \rceil !.$$

Lower Bound of $\mathrm{Pa}(n, P)$

By an elaborate extension of what we saw for $P = \mathcal{V}$, we construct an $\mathcal{F}_n \subseteq \mathcal{B}_n$, of unrelated copies of P, where as $n \to \infty$,

$$|\mathcal{F}_n| \sim |P| \sum_{j=0}^{\infty} \left(\frac{(2^k - c(P))^j}{(2^k)^{j+1}} \right) \binom{n}{\lfloor \frac{n}{2} \rfloor}$$

$$= |P| \frac{1}{2^k} \left[\frac{1}{1 - \frac{2^k - c(P)}{2^k}} \right] \binom{n}{\lfloor \frac{n}{2} \rfloor} = \frac{|P|}{c(P)} \binom{n}{\lfloor \frac{n}{2} \rfloor},$$

which gives the Theorem.

Packing Induced Copies of P

Denote by $\mathrm{Pa}^*(n, \{P_i\})$ the maximum size of a family $\mathcal{F} \subseteq \mathcal{B}_n$, where each connected component is an *induced* copy of a poset from the collection $\{P_i\}$.

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Theorem (D., G. (2013) and Katona, Nagy (2013))
As $n \to \infty$, $\mathrm{Pa}^*(n, P) \sim \frac{|P|}{c^*(P)} \binom{n}{\lfloor \frac{n}{2} \rfloor}$.

Other Results

For a finite collection of posets:

Theorem (D., G. (2013))

As n goes to infinity,

$$\mathrm{Pa}(n, \{P_1, P_2, \ldots, P_k\}) \sim \max_{1 \le i \le k} \left(\frac{|P_i|}{c(P_i)} \right) \binom{n}{\lfloor \frac{n}{2} \rfloor}.$$

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- Finding $\mathrm{Pa}(n, \{P_i\})$ asymptotically for an infinite collection of posets.
- Finding exact values of $\mathrm{Pa}(n, P)$.
- Designing an algorithm that quickly finds $c(P)$, or even the complexity of such an algorithm.


Thank You