Math 546 Exam 3 Syllabus
April, 2010

1. Introduction to Groups
   a) Binary operations; closure; associative and commutative properties
   b) Full definition of a group (all four axioms defined)
   c) Uniqueness of the identity and of inverses +pfs
   d) Semigroups
   f) Abelian groups and related examples and exercises
   g) Operation tables, Latin square property for groups
   i) Cancellation laws +pf and solving equations in groups

2. Group Theory Basics
   a) Order of a group \( o(G) = |G| \); order \( o(g) \) of an element \( g \in G \)
   b) Subgroups defn. and Subgroup Theorem +pf
   c) \( \langle g \rangle \); Cyclic groups
   n) Lagrange’s Theorem (+pf) and applications
   o) The intersection of groups
   p) Exercises on groups (homework assignments)

3. Examples of Groups
   a) \( \mathbb{Z}, \mathbb{Z}_n, \mathbb{Q}, \mathbb{R} \) under +
   b) \( a\mathbb{Z} \) under + for integer \( a \)
   c) Symmetric group \( S_n \) of permutations on \( \{1, \ldots, n\} \)
   d) Alternating group \( A_n \) of even permutations in \( S_n \)
   f) Matrices \( M_n(\mathbb{R}) \) under addition
   g) General linear group \( GL_n(F) \) of invertible matrices of order \( n \) over \( F \), where field \( F \) is \( \mathbb{R}, \mathbb{Q}, \) or \( \mathbb{Z}_2 \)
   h) Semigroup of functions under composition
   i) Semigroup of words on an alphabet under concatenation
   j) Groups of order \( \leq 6 \) including Klein group \( V \) \( (V_4) \)
   k) Quaternion group of order 8
   n) The group of units \( \mathbb{Z}_n^\times \) under \( \times \)