Algebraic Structures I
Math 546 & 701I
Spring, 2009
Course Syllabus

Professor: Jerry Griggs
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Off. Hrs.: LeConte 409 (x-7-4225); M 2:30-4, Th 2-3:30 (tent.).

Feel free to drop by and chat; I’m often available in my office, especially most afternoons, and will meet with you if I can. I’d be happy to make an appointment with you!


Background: I am assuming some prior introduction to "higher mathematics", including the logic of mathematical statements, basic facts about sets, proofs by induction, matrix multiplication and determinants. Students are best prepared for the course who have already completed Math 574 (Discrete Math) and either Math 526 or 544 (Linear Algebra). Our text includes some of this necessary background in detail in the Appendix.

Overview: This first course on algebraic structures concentrates on basic examples and theory about groups.

Groups arise in both discrete and continuous settings where symmetry is involved, ranging from analysis and topology to enumeration, crystallography, pattern design (Escher), data transmission (coding theory), and quantum physics. Their applications include the nonconstructability of certain geometric objects, the nonsolvability of certain equations, and cryptography. Deeper results from group theory along with a thorough study of related structures (rings and fields) are developed in the sequel course, Math 547 (next offered in Spring, 2010).

Outcomes: The goal of the course is for students to learn what groups are, to be able to recognize them, and be familiar with the fundamental examples of groups. Students will be expected to learn and state the main definitions and theorems in the course. Students will become proficient at writing proofs to derive basic properties of groups and to solve problems which are particular statements about groups.

Students will first master the fundamental number-theoretic material about the integers and divisibility, including the Quotient-Remainder Theorem and modular arithmetic (Chapter 1).

We will continue with additional basic material, learning what equivalence relations are and their characterization in terms of partitions of sets. We shall next learn about the permutations of an n-element set, how to multiply permutations, and how to decompose permutations into cycles (Chapter 2).

Armed with these examples, we consider the notion of groups: what they are, and how to recognize them (Chapter 3). We shall learn what is the order of an element of a group. We shall learn all groups of small order and the families of cyclic groups and of
permutation groups. We shall learn about the direct products of groups. We shall analyze
the symmetries of basic geometric patterns and objects. As time permits, we shall learn
about the mappings between groups called group homomorphisms and their application to
studying the structure of specific groups (cosets, normal subgroups, and factor groups).

Grading:

Exams: There will be three midterm exams for the course, and a comprehensive final
exam (at 2pm on May 2). Each exam will be assigned letter grades on a separate curve
based on the difficulty of the particular test.

Each midterm exam is worth 100 points, and the final is worth 150 points. The sum
of the scores (out of a possible 450) will be compared to the sums of the grade cutoffs to
determine the final grade. There is no grade breakdown in advance—if all students excel,
all will earn high grades—which would be terrific.

Classroom participation and any bonus assignment credit will be also be considered
in deciding the final grade.

Graduate students (in Math 7011) have an additional requirement: Each graduate
student will also complete a project, which can be a report about an application of group
theory (such as the topics mentioned above), or the design of some interesting models
related to group theory, or solutions of some challenging exercises. Other proposals will
be welcomed. All sources must be carefully cited; your report needs to be original. The
due date is April 20.

Homework: Homework will be assigned and spot-checked, though not scored. It is
very important to work out many homework exercises, though time and resources make
it impossible to check them all for credit. However, strong performance on homework will
be viewed positively!

Policies: Regular attendance is essential for success in math classes. So attendance will be
checked at each class. Absences in excess of 10 percent of the classes (five or more) can
affect the final grade—please talk to me if you anticipate missing this many classes.

Students are encouraged to form study groups to go over the course material and
elements. Please be inclusive of class members.

It is important that every answer be justified by showing enough reasoning or com-
putations so that someone else can understand how it was obtained. When possible, one
should answer with complete sentences. I want you to make progress in communicating
mathematical ideas.