Exercise. Let f be entire. Show if f is not a constant function, then $f(\mathbb{C})$ be dense in $\mathbb{C}$.
Hint: We could have done this exercise awhile back but it now makes a nice comparison with the Casorati-Weirstrass Thm. (Thm III.2.6: z_0 essential sing. $\Rightarrow f(B'_\varepsilon(z_0))$ is dense is $\mathbb{C}$).

Proof Sketch. Let f be entire. We will show that if $f(\mathbb{C})$ is not dense in $\mathbb{C}$, the f is constant.

If $f(\mathbb{C})$ is not dense in $\mathbb{C}$, then there is a disk $D(z_0, r)$ such that $D(z_0, r) \cap f(\mathbb{C}) = \emptyset$. Thus for all $z \in \mathbb{C}$, $|f(z) - z_0| \geq r$, and the result now follows from Liouville's theorem applied to $1/[f(z) - z_0]$.