**Exercise.**
- Review Logarithm Functions handout, which is posted on the course homepage but also linked for your convenience.
- Let the path $\gamma$ be join of the three line segments: $[1-i, 1+i]$ and $[1+i, -1+i]$ and $[-1+i, -1-i]$, as indicated below in Figure 1. Evaluate

$$\int_\gamma \frac{dz}{z}$$

by using an appropriate branch of $\log z$.
Be sure to specify the branch (or equiv., branch cut) you use.

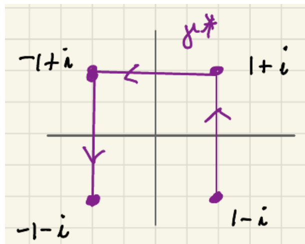


FIGURE 1. $\gamma^*$

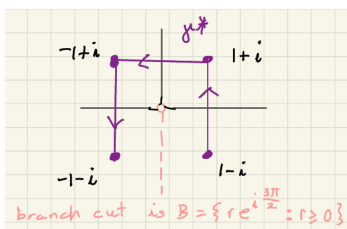


FIGURE 2. branch cut

LTGBG . For the branch cut $B$ of the logarithm function take

$$B := \left\{ re^{i\frac{3\pi}{2}} \in \mathbb{C} \colon r \geq 0 \right\} \overset{\text{i.e.}}{=} \{z \in \mathbb{C} \colon \operatorname{Re} z = 0 \text{ and } \operatorname{Im} z \leq 0\}$$

and consider

$$G := \mathbb{C} \setminus B \overset{\text{note}}{=} \left\{ re^{i\theta} \in \mathbb{C} \colon r > 0 \,,\ \frac{-\pi}{2} < \theta < \frac{3\pi}{2} \right\}.$$

Thus $G$ is an open connected set containing $\gamma^*$. Let $f \colon G \setminus B \to \mathbb{C}$ be the branch of the log on $G$ given by, for $z \in \mathbb{C} \setminus B$, ⟨here $\ln \colon (0, \infty) \to \mathbb{R}$ is the (real-variables) natural log function⟩

$$f(z) = \ln|z| + i \arg z \qquad \text{where} \quad \frac{-\pi}{2} < \arg z < \frac{3\pi}{2}.$$

By Cor I.3.16 ⟨p. 8⟩, $f \in H(G)$ and,

$$\text{for each } z \in G, \qquad f'(z) = \frac{1}{z}.$$

By the FTC for path integrals ⟨Thm II.2.8, p17⟩, since $f$ is holomorphic on (an open set containing) $\gamma^*$ and $f'$ is continuous on $\gamma^*$,

$$\int_\gamma \frac{dz}{z} = \int_\gamma f'(z)\, dz \overset{\text{FTC}}{=} f(-1-i) - f(1-i) = f\left(\sqrt{2}e^{i\frac{5\pi}{4}}\right) - f\left(\sqrt{2}e^{i\frac{-\pi}{4}}\right)$$

$$= \left[\ln\sqrt{2} + i\frac{5\pi}{4}\right] - \left[\ln\sqrt{2} + i\frac{-\pi}{4}\right] = i\frac{6\pi}{4} = \boxed{\frac{3\pi\, i}{2}}.$$