

You learn a lot talking math with others. Thus you are **strongly** encouraged to work in groups (up to size 17) on homework. A group is to come to an agreement of the finished paper. Over Blackboard, ONE group member (e.g., Bella) should submit the finished paper while each of the other group members (as so that I can return a commented graded paper to you) should just pull up the assignment on Blackboard and write a note in the white **Comment** box that, e.g., Bella submitted my paper.

Metric Space Exercise 7. Variant of 2.1.45.22 (p. 93).

Let A and B be non-empty subsets of a metric space (X, d) . Prove the following. You may use anything from class and/or the Metric Spaces two page handout.

Metric Space Exercise 7i.

The set A is bounded if and only if there exist $x \in X$ and $r > 0$ such that $A \subset B_r(x)$.

Metric Space Exercise 7ii. $A \subset B$ implies that $\text{diam } A \leq \text{diam } B$.

Metric Space Exercise 7iii. $\text{diam } A = 0$ if and only if for some $x \in X$ we have that $A = \{x\}$.

Metric Space Exercise 7iv. If $a \in A$ and $b \in B$, then

$$\text{diam } (A \cup B) \leq \text{diam } (A) + \text{diam } (B) + d(a, b).$$

Metric Space Exercise 7v. If A and B are bounded, then $A \cup B$ is bounded; furthermore, a finite union of bounded subsets of X is bounded.