

You learn a lot talking math with others. Thus you are **strongly** encouraged to work in groups (up to size 17) on homework. A group is to come to an agreement of the finished paper and then each group member should submit over Blackboard the identical finished paper. Follow the instructions at the top of the LaTeX file to put all PINs and Names on the paper. A graded copy of the group's finished paper will be returned to each group member.

Metric Space Exercise 3. Variant of 2.1.45.4 (p. 90)

Let S be the set of all sequences of real numbers and define $d: S \times S \rightarrow \mathbb{R}$ by

$$d(x, y) := \sum_{n=1}^{\infty} \frac{|x_n - y_n|}{2^n [1 + |x_n - y_n|]}, \quad x = \{x_n\}_{n=1}^{\infty} \in S \text{ and } y = \{y_n\}_{n=1}^{\infty} \in S. \quad (1)$$

Show that (S, d) is a metric space.

Hint: you may use, without proving, that the function $f: [0, \infty) \rightarrow \mathbb{R}$ given by $f(t) := \frac{t}{1+t}$ is a strictly increasing function.