Exercise.
Let $f \in H(B_1(0))$ satisfy that

(i) $|f(z)| \leq 1$ for each $z \in B_1(0)$
(ii) $f(0) = 0$.

Show that

(a) $|f(z)| \leq |z|$ for each $z \in B_1(0)$,
(b) $|f'(0)| \leq 1$.

If, furthermore, $|f(z_0)| = |z_0|$ for some $z_0 \in B_1'(0)$, show that

(c) there exists $c \in \mathbb{C}$ with $|c| = 1$ such that $f(z) = cz$ for each $z \in B_1(0)$.

Recall: $B_1(0) := \{z \in \mathbb{C} \colon \ |z| < 1\}$.
Remark: this exercise is known as Schwarz's Lemma.