Exercise. Problem Source: Quals 1995.
Let $f \in H(\mathbb{C})$ satifsy, for some constants $A, B \in \mathbb{R}$ and $k \in \mathbb{N}$,

$$|f(z)| \leq A|z|^k + B \tag{1}$$

for each $z \in \mathbb{C}$. Prove that f is a polynomial.
Hint: use the CIF (see Cor. II.2.24a from class).