$\boxed{\text{Recall}}$

Cauchy-Riemann Equations for $f = u + iv$ are: $u_x = v_y$ and $u_y = -v_x$.

Prop. 4.10. If $f \in H(G)$ and $f'(z) = 0$ for each z in the nonempty open connected subset G of $\mathbb{C}$, then f is constant on G.

Exercise. Let $f \in H(G)$ where G is a nonempty open connected subset of $\mathbb{C}$. Prove the following.
1. If $\operatorname{Re} f$ is constant on G, then f is constant on G.
2. If $\operatorname{Im} f$ is constant on G, then f is constant on G.
3. If $|f|$ is constant on G, then f is constant on G.

Do so without using facts not covered thus far in class. So you may use ideas from the Class Script's Section 1.1-1.3 as well as Prop. 4.10.