Exercise. Prove the following functions are nowhere (complex) differentiable on $\mathbb{C}$.
Recall that (complex) differentiable means differentiable as defined in Class Script, p. 4, Def. I.3.1.

(a) $f(z) = \operatorname{Re} z$.

(b) $f(z) = |z|$.

Recall that (complex) differentiable means differentiable as defined in Class Script, p. 4, Def. I.3.1.
Def. A function $f\colon G \to \mathbb{C}$, where G is an open subset of $\mathbb{C}$, is nowhere differentiable on G provided f is not differentiable at z for each $z \in G$.
Remark. As usual in such a setting, we write $z = x+iy$ with $x, y \in \mathbb{R}$ and let $u(x,y) = \operatorname{Re}(f(x,y))$ and $v(x,y) = \operatorname{Im}(f(x,y))$. Thus $f(x+iy) = u(x,y) + iv(x,y)$.

Recall. The Cauchy-Riemann Equations (CReq) for f are

$$u_x = v_y \qquad \text{and} \qquad u_y = -v_x \ . \tag{CReq}$$

Recall some Big Theorems

Let $f\colon G \to \mathbb{C}$ where G is an open subset of $\mathbb{C}$. Fix $z_0 \in G$.

▷ Diff⇒CR. Let f is differentiable at z_0. Then the CR equations for f are satisfied at z_0.

▷ CR+more⇒Diff. Let the CR equations for f be satisfied at z_0 and, furthermore, the first partial derivatives u_x, v_y, u_y, and v_x:

1. exist in some neighborhood $N_\varepsilon(z_0)$ of z_0
2. be continuous at z_0.

Then f is differentiable at z_0.

▷ If f is differentiable at $z_0 = x_0 + iy_0$, then $f'(z_0) = u_x(x_0, y_0) + iv_x(x_0, y_0) = v_y(x_0, y_0) - iu_y(x_0, y_0)$.
