Hint. These Exercises use heavily the following properties of outer measure $|\cdot| : \mathcal{P}(\mathbb{R}) \to [0, \infty]$.

2.5. Outer measure preserves order (i.e., if $A \subset B \subset \mathbb{R}$ then $|A| \leq |B|$).

2.8. Outer measure is countably subadditive (i.e., if $\{A_n\}_{n\in\mathbb{N}}$ is a sequence of subsets of $\mathbb{R}$, then $\left|\bigcup_{k=1}^{\infty} A_k\right| \leq \sum_{k=1}^{\infty} |A_k|$).

Exercise 2A1. Prove that if A and B are subsets of $\mathbb{R}$ and $|B| = 0$, then $|A \cup B| = |A|$.

Proof. please put your proof of 2A1 here ☐

Exercise 2A6. Prove that if $a, b \in \mathbb{R}$ and $a < b$, then

$$|(a, b)| = |[a, b)| = |(a, b]| = b - a.$$

Proof. please put your proof of 2A6 here ☐

Exercise 2A10. Prove that $|[0, 1] \setminus \mathbb{Q}| = 1$.

Proof. please put your proof of 2A10 here ☐