In this handout,

$$A \text{ and } B \text{ are nonempty bounded subsets of } \mathbb{R} \qquad \text{and} \qquad r \in \mathbb{R}\ .$$

## Notation

$$\begin{aligned}
{}^{-}A &:= \{-a \in \mathbb{R} \colon a \in A\} \\
r + A &:= \{r + a \in \mathbb{R} \colon a \in A\} \\
rA &:= \{ra \in \mathbb{R} \colon a \in A\} \\
A + B &:= \{a + b \in \mathbb{R} \colon a \in A, b \in B\} && \text{(called the Minkowski sum)} \\
AB &:= \{ab \in \mathbb{R} \colon a \in A, b \in B\} && \text{(Not commonly used notation but let's use for this handout.)}
\end{aligned}$$

## Results for Max/Min

If $\max A$ exists, then $$\min\left({}^{-}A\right) = {}^{-}(\max A)\ . \qquad (1)$$

If $\min A$ exists, then $$\max\left({}^{-}A\right) = {}^{-}(\min A)\ . \qquad (2)$$

## Results for Sup/Inf

$$\inf\left({}^{-}A\right) = {}^{-}(\sup A) \qquad (3)$$

$$\sup\left({}^{-}A\right) = {}^{-}(\inf A) \qquad (4)$$

$$\inf(r + A) = r + (\inf A) \qquad (5)$$

$$\sup(r + A) = r + (\sup A) \qquad (6)$$

$$\inf(A + B) = (\inf A) + (\inf B) \qquad (7)$$

$$\sup(A + B) = (\sup A) + (\sup B) \qquad (8)$$

for $r \geq 0$: $$\inf(rA) = r(\inf A) \qquad (9)$$

for $r \geq 0$: $$\sup(rA) = r(\sup A) \qquad (10)$$

for $r \leq 0$: $$\inf(rA) = r(\sup A) \qquad (11)$$

for $r \leq 0$: $$\sup(rA) = r(\inf A) \qquad (12)$$

for $A, B \subset [0, \infty)$: $$\inf(AB) = (\inf A)(\inf B) \qquad (13)$$

for $A, B \subset [0, \infty)$: $$\sup(AB) = (\sup A)(\sup B) \qquad (14)$$