Example. The universe is $\mathbb{R}$ and index set is $\mathbb{N}$.

Let $\{A_n\}_{n\in\mathbb{N}}$ be a collection of subset of $\mathbb{R}$. Then we have the following subsets of $\mathbb{R}$.

Recall for the two sets

$$A_1 \cup A_2 \overset{\text{say}}{=} \text{ union of } A_1 \text{ and } A_2 \overset{\text{def}}{:=} \{x \in \mathbb{R} \colon x \in A_1 \text{ or } x \in A_2\}$$
$$\overset{\text{i.e.}}{=} \{x \in \mathbb{R} \colon x \text{ is in (at least) one of the two sets: } A_1,\ A_2\}.$$

Also can write as: $A_1 \cup A_2 = \bigcup_{n=1}^{2} A_n = \bigcup_{n\in\{1,2\}} A_n$.

When the index set is $\mathbb{N}$,

$$\bigcup_{n\in\mathbb{N}} A_n \overset{\text{i.e.}}{=} \bigcup_{n=1}^{\infty} A_n \overset{\text{def}}{:=} \{x \in \mathbb{R} \colon x \text{ is in (at least) one of the sets from } \{A_n\}_{n\in\mathbb{N}}\}.$$

Recall for two sets

$$A_1 \cap A_2 \overset{\text{say}}{=} \text{ intersection of } A_1 \text{ and } A_2 \overset{\text{def}}{:=} \{x \in \mathbb{R} \colon x \in A_1 \text{ and } x \in A_2\}$$
$$\overset{\text{i.e.}}{=} \{x \in \mathbb{R} \colon x \text{ is in each of the two sets: } A_1,\ A_2\}$$

Also can write as: $A_1 \cap A_2 = \bigcap_{n=1}^{2} A_n = \bigcap_{n\in\{1,2\}} A_n$.

When the index set is $\mathbb{N}$,

$$\bigcap_{n\in\mathbb{N}} A_n \overset{\text{i.e.}}{=} \bigcap_{n=1}^{\infty} A_n \overset{\text{def}}{:=} \{x \in \mathbb{R} \colon x \text{ is in each of the sets from } \{A_n\}_{n\in\mathbb{N}}\}.$$

For an arbitrary universal set U and an arbitary index set I.

⟨Warning. The univeral set U and index set I serve different purposes.⟩ Let $\{A_i\}_{i\in I}$ be a collection of subset of U. The union of $\{A_i\}_{i\in I}$ over the index set I:

$$\bigcup_{i\in I} A_i \overset{\text{def.}}{=} \{x \in U \colon x \text{ is in (at least) one of the } A_i\text{'s}\}$$
$$\overset{\text{i.e.}}{=} \{x \in U \colon \text{there exists an } i \in I \text{ such that } x \in A_i\} \overset{\text{note}}{\subseteq} U.$$

The intersection $\{A_i\}_{i\in I}$ over the index set I:

$$\bigcap_{i\in I} A_i \overset{\text{def.}}{=} \{x \in U \colon x \text{ is in each of the } A_i\text{'s}\}$$
$$\overset{\text{i.e.}}{=} \{x \in U \colon\ x \in A_i \text{ for each } i \in I\} \overset{\text{note}}{\subseteq} U.$$

Symbolically,

$$\left[x \in \bigcup_{i\in I} A_i \right] \Leftrightarrow \left[(\exists i \in I)\ [x \in A_i] \right] \quad \text{while} \quad \left[x \in \bigcap_{i\in I} A_i \right] \Leftrightarrow \left[(\forall i \in I)\ [x \in A_i] \right].$$

So when $I = \emptyset$.

$$\bigcup_{i\in\emptyset} A_i = \emptyset \qquad \text{while} \qquad \bigcap_{i\in\emptyset} A_i = U.$$