Ex 1. Which of the following sets S are fields (resp. ordered fields, complete ordered fields) when endowed with the usual addition operation, muliplication operation, and order?

Fill in the boxes with YES or NO. No proof needed, just use intuition. If answer NO, give a reason.

S $\Downarrow$	English for S	description of S	field	ordered field	complete ordered field
$\mathbb{N}$	natural numbers	$\{1,2,3,4,\ldots\}$	No. lacks additive identity	No. not a field	No. not a field
$\mathbb{Z}$	integers	$\{\ldots,-2,-1,0,1,2,\ldots\}$	No. lacks mult. inverses	No. not a field	No. not a field
$\mathbb{Q}$	rational numbers	$\{\frac{a}{b}\in\mathbb{R}\colon a\in\mathbb{Z}, b\in\mathbb{N}\}$ hint: consider $\left[0,\sqrt{2}\right)\cap\mathbb{Q}$	Yes.	Yes.	No.
$\mathbb{R}$	real numbers	the unique complete ordered field	Yes. by definition	Yes. by definition	Yes. by definition

Ex 2. Consider the below subsets S of $\mathbb{R}$. Find the following, when they exist. Just use your intuition. No proofs needed. Use $\nexists$ for does not exists. Columns B and E answers may vary. For intuition, think of:

- lub S = least upper bound of S = supremum of S = $\sup S$
- glb S = greatest lower bound of S = infimum of S = $\inf S$.

	order $\Rightarrow$	D	E	F	H	A	B	C	G
	S $\Downarrow$	Is S bounded below?	some lower bounds of S	$\min S$	glb S = $\inf S$	Is S bounded above?	some upper bounds of S	$\max S$	lub S = $\sup S$
	recall $S\subset\mathbb{R}$	yes/no	must be in $\mathbb{R}$	must be in S	in $\widehat{\mathbb{R}}:=\mathbb{R}\cup\{\pm\infty\}$	yes/no	must be in $\mathbb{R}$	must be in S	in $\widehat{\mathbb{R}}:=\mathbb{R}\cup\{\pm\infty\}$
2.1	$\{-3,2,5,7\}$	yes	-3, -17	-3	-3	yes	7, 17	7	7
2.2	$[\,0,\sqrt{2}\,]$	yes	0, -17	0	0	yes	$\sqrt{2}$, 17	$\sqrt{2}$	$\sqrt{2}$
2.3	$[\,0,17\,)$	yes	0, -17	0	0	yes	17, 27	$\nexists$	17
2.4	$\mathbb{R}$	no	$\nexists$	$\nexists$	$-\infty$	no	$\nexists$	$\nexists$	$+\infty$
2.5	$\emptyset$	yes	-17, 0, 17	$\nexists$	$+\infty$	yes	-17, 0, 17	$\nexists$	$-\infty$
2.6	$\{x\colon x^3<8\}$	no	$\nexists$	$\nexists$	$-\infty$	yes	2, 17	$\nexists$	2
2.7	$\{\frac{1}{n}\colon n\in\mathbb{N}\}$	yes	0, -17	$\nexists$	0	yes	1, 17	1	1
2.8	$\{\frac{1}{x}\colon x\in\mathbb{R}^{>0}\}$	yes	0, -17	$\nexists$	0	no	$\nexists$	$\nexists$	$+\infty$
2.9	$\{\cos n\pi\colon n\in\mathbb{N}\}$	yes	-1, -17	-1	-1	yes	1, 17	1	1
2.10	$\{n\cos n\pi\colon n\in\mathbb{N}\}$	no	$\nexists$	$\nexists$	$-\infty$	no	$\nexists$	$\nexists$	$+\infty$
2.11	$\mathbb{N}$	yes	1, 0, -17	1	1	no	$\nexists$	$\nexists$	$+\infty$
2.12	$\mathbb{Z}$	no	$\nexists$	$\nexists$	$-\infty$	no	$\nexists$	$\nexists$	$+\infty$
2.13	$\mathbb{Q}$	no	$\nexists$	$\nexists$	$-\infty$	no	$\nexists$	$\nexists$	$+\infty$