

On this homework (provided your specifically reference it when used) you may use the following Theorem from class.

**Nested Interval Property.** (Really a Theorem).

Let  $(I_n)_{n=1}^{\infty}$  be a sequence of nonempty closed bounded intervals of  $\mathbb{R}$  that are nested in the sense

$$I_{j+1} \subseteq I_j \quad \text{for each } j \in \mathbb{N}.$$

Then

(1)  $\bigcap_{n=1}^{\infty} I_n$  is nonempty,

(2) and if furthermore the  $\lim_{n \rightarrow \infty} \text{length}(I_n) = 0$  then  $\bigcap_{n=1}^{\infty} I_n$  has precisely one element.

**Problem 39**. Variant of Exercise 2.5.9.

§2.5  
 BS4p52

1. For each  $n \in \mathbb{N}$ , let

$$K_n := [n, \infty).$$

Prove that  $\bigcap_{n=1}^{\infty} K_n = \emptyset$ .

2. In the above statement of the **Nested Interval Property**, can the assumption that the intervals are bounded be omitted? Justify your answer. BTW: each  $K_n$  is closed (see top of p. 47).

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