Recall the picture for the Cantor Diagonalizational Argument we used to show $\mathbb{R}$ is uncountable.

$$\begin{aligned} x^1 &= 0.d_1^1\ d_2^1\ d_3^1\ d_4^1\ d_5^1 \cdots \\ x^2 &= 0.d_1^2\ d_2^2\ d_3^2\ d_4^2\ d_5^2 \cdots \\ x^3 &= 0.d_1^3\ d_2^3\ d_3^3\ d_4^3\ d_5^3 \cdots \\ x^4 &= 0.d_1^4\ d_2^4\ d_3^4\ d_4^4\ d_5^4 \cdots \end{aligned} \tag{CDA}$$

Problem 37.Not in book. §2.5 BS4

1. Let A be the set of all *finite* sequences of 0's and 1's. To clarify,

$$A \;=\; \left\{ \; (a_i)_{i=1}^{n} : n \in \mathbb{N} \text{ and } a_i \in \{0,1\} \text{ for each } i \in \mathbb{N}^{\leq n} \; \right\}$$

Prove that A is countably infinite.

HINT. Ideas from Problem 19, in which we showed that the collection of all *finite* subsets of $\mathbb{N}$ is denumerable, might be helpful.

2. Let B be the set of all *infinite* sequences of 0's and 1's. To clarify,

$$B \;=\; \{ \; (b_i)_{i=1}^{\infty} : b_i \text{ is 0 or 1 for each } i \in \mathbb{N} \; \}$$

Prove that B is uncountable by using a Cantor Diagonalizational Argument.

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