On this homework, specifically say where you are using Archimedean's Property (or one of it's corollaries) when you use it. Below is a list of the versions of Archimedean's Property that we showed in class.
Thm. (Archimedean's Property) $(\forall b \in \mathbb{R})\ (\forall a \in \mathbb{R}^{>0})\ (\exists n \in \mathbb{N})\ [b < na]$
Cor. 1. $(\forall x \in \mathbb{R})\ (\exists n \in \mathbb{N})\ [x < n]$
Cor. 2. $(\forall \epsilon > 0)\ (\exists n \in \mathbb{N})\ \left[\frac{1}{n} < \epsilon\right]$
Cor. 3. $(\forall z \in \mathbb{R}^{>0})\ (\exists n \in \mathbb{N})\ [n - 1 \leq z < n]$

$\boxed{\mathbf{36}}$. Variant of Exercise 2.4.19. §2.4 BS4p46

Let $x, y, p \in \mathbb{R}$ such that

$$x < y \tag{1}$$

and

$$p > 0. \tag{2}$$

Show (i.e. prove) that there exists a rational number r such that

$$x < rp < y. \tag{3}$$

Remark. Compare with ⟨from p. 44⟩ the book's 2.4.8 (which says $\mathbb{Q}$ is dense in $\mathbb{R}$) and 2.4.9 (which says the irrational numbers are dense in $\mathbb{R}$). This problem says the set $\{rp \colon r \in \mathbb{Q}\}$ is dense in $\mathbb{R}$.

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