On this homework, specifically say where you are using Archimedean's Property (or one of it's corollaries) when you use it. Below is a list of the versions of Archimedean's Property that we showed in class.
Thm. (Archimedean's Property) $(\forall b \in \mathbb{R})\ (\forall a \in \mathbb{R}^{>0})\ (\exists n \in \mathbb{N})\ [b < na]$
Cor. 1. $(\forall x \in \mathbb{R})\ (\exists n \in \mathbb{N})\ [x < n]$
Cor. 2. $(\forall \epsilon > 0)\ (\exists n \in \mathbb{N})\ \left[\frac{1}{n} < \epsilon\right]$
Cor. 3. $(\forall z \in \mathbb{R}^{>0})\ (\exists n \in \mathbb{N})\ [n - 1 \leq z < n]$

On this homework, you may use the below lemmas. Hints on their proofs, which are straightforward, are given so you do not have to prove the lemmas.
Lemma 1. If $n \in \mathbb{N}$, then $n < 2^n$. Idea of proof. Use math induction.
Lemma 2. If $x \in [1, \infty)$, then $x + 1 \leq 2^x$. Idea of proof. Use the Race Track Principle with the ⟨slow⟩ function $s(x) := x + 1$ and the ⟨fast⟩ function $f(x) := 2^x$ with $s, f \colon [1, \infty) \to \mathbb{R}$. Recall, $f'(x) = (\ln 2)\, 2^x$.

35. Variant of Exercise 2.4.14. §2.4 BS4p46

Let $\epsilon > 0$. Prove that there exists $n \in \mathbb{N}$ such that

$$\frac{1}{2^n} < \epsilon.$$

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