Put your Pin and Name into the LaTeX template (see first few lines) as well as on back of the last page in the upper right corner.
Turn in a stapled hard copy (copied one-sided) of your homework (folded to 4.25x11 size) at the beginning of class
If you need to use a staper, there is a stapler outside my office (1011c) door for your convenience.

On this homework, specifically say where you are using Archimedean's Property (or one of it's corollaries) when you use it. Below is a list of the versions of Archimedean's Property that we showed in class.
Thm. (Archimedean's Property) $(\forall b \in \mathbb{R})\ (\forall a \in \mathbb{R}^{>0})\ (\exists n \in \mathbb{N})\ [b < na]$
Cor. 1. $(\forall x \in \mathbb{R})\ (\exists n \in \mathbb{N})\ [x < n]$
Cor. 2. $(\forall \epsilon > 0)\ (\exists n \in \mathbb{N})\ \left[\frac{1}{n} < \epsilon\right]$
Cor. 3. $(\forall z \in \mathbb{R}^{>0})\ (\exists n \in \mathbb{N})\ [n - 1 \le z < n]$

28. Variant of Exercise 2.4.1. §2.4 BS4p44

Prove that $\sup\left\{1 - \dfrac{1}{n} : n \in \mathbb{N}\right\} = 1$.

29. Variant of Exercise 2.4.2. §2.4 BS4p44

Conjecture the $\sup\left\{\dfrac{1}{j} - \dfrac{1}{k} : j, k \in \mathbb{N}\right\}$. Then prove your conjecture is true.

30. Variant of Exercise 2.4.2. §2.4 BS4p44

Conjecture the $\inf\left\{\dfrac{1}{j} - \dfrac{1}{k} : j, k \in \mathbb{N}\right\}$. Then prove your conjecture is true.

31. Variant of Exercise 2.4.4. For $r \in \mathbb{R}$ and $B \subseteq \mathbb{R}$, define $rB := \{rb : b \in B\}$. §2.4 BS4p45

Let T be a nonempty bounded set in $\mathbb{R}$ and $p > 0$. Prove that $\sup(pT) = p \sup T$.

32. Variant of Exercise 2.4.4. For $r \in \mathbb{R}$ and $B \subseteq \mathbb{R}$, define $rB := \{rb : b \in B\}$. §2.4 BS4p45

Recall from class: **Lemma 1**. Let $B \subset \mathbb{R}$. Then $\sup(-B) = -\inf B$.

Thm. 2. Let T be a nonempty bounded set in $\mathbb{R}$ and $n < 0$. ⟨Note $n \in \mathbb{R}$.⟩ Then $\sup(nT) = n \inf T$.

Give a (short) proof of Thm. 2 by using Lemma 1 and the previous problem.

HINT: $nT = (-n)(-T)$.

33. Variant of Exercise 2.4.8. §2.4 BS4p45

Let X be a nonempty set. Let the function $f : X \to \mathbb{R}$ and $g : X \to \mathbb{R}$ have bounded ranges.

Prove

$$\sup\{f(x) + g(x) : x \in X\} \ \le\ \sup\{f(x) : x \in X\} \ +\ \sup\{g(x) : x \in X\}. \tag{1}$$

Also give an example showing that the inequality in (1) need not be an equality.