Put your Pin and Name into the LaTeX template (see first few lines) as well as on back of the last page in the upper right corner.
Turn in a stapled hard copy (copied one-sided) of your homework (folded to 4.25x11 size) at the beginning of class
If you need to use a staper, there is a stapler outside my office (1011c) door for your convenience.

24. Exercise 2.2.101. Not in book. Triangle Inequality. §2.2 BS4p32

The following facts are proved in the book.

Triangle Inequalities. Let $x, y \in \mathbb{R}$. Then

$$|x+y| \le |x| + |y| \tag{Thm. 2.2.3}$$

$$|x-y| \le |x| + |y| \tag{Cor. 2.2.4b}$$

$$|\,|x| - |y|\,| \le |x-y| \tag{Cor. 2.2.4a}$$

Using the above 3 triangle inequalities ⟨which are proved in the book⟩, prove that (a fourth triangle inequality)

$$|\,|x| - |y|\,| \le |x+y| \tag{24}$$

Remark. The purpose of this problem is to get all four Triangle Inequalities; indeed, **after** proving the inequality in (24), we can combine (24) with (Thm. 2.2.3), (Cor. 2.2.4a), and (Cor. 2.2.4b) to get the four inequalites:

$$\boxed{\,|\,|x| - |y|\,| \le |x \pm y| \le |x| + |y|\,}\,. \tag{$\triangle$-inequalities}$$

Cut this out and then put your answer here.

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25. Exercise 2.3.101. Not in book. Max/Min and Sup/Inf chart. §2.3 BS4

Consider the below subsets S of $\mathbb{R}$. Find the $\max S$ and $\min S$, provided they exist (use $\nexists$ for does not exists). Find the $\sup S$ and $\inf S$ in the extended sense (so in $\widehat{\mathbb{R}} \overset{\text{i.e.}}{=} \mathbb{R} \cup \{\pm\infty\}$). No proofs needed.

Just use your intuition. Just type your solutions directly into the below chart. Some samples from class are included.

	S $\Downarrow$	$\min S$	$\inf S$	$\max S$	$\sup S$
recalls $\Rightarrow$	$S \subseteq \mathbb{R}$	must be in S	in $\widehat{\mathbb{R}}$	must be in S	in $\widehat{\mathbb{R}}$
Samples from class.					

sample	$[0, \sqrt{2}]$	0	0	$\sqrt{2}$	$\sqrt{2}$
sample	$\mathbb{R}$	$\nexists$	$-\infty$	$\nexists$	$+\infty$
Now onto the assigned homework problems.					
(1)	$\{x \colon x^2 - 4x - 5 \leq 0\}$				
(2)	$\left\{1 + \frac{(-1)^n}{n} : n \in \mathbb{N}\right\}$				
(3)	$(0,1) \cap \mathbb{Q}$				

$\boxed{\textbf{26}}$. Variant of Exercise 2.3.10. §2.3 BS4p40

Let A and B be nonempty bounded subsets of $\mathbb{R}$.

(1) Is $A \cup B$ is a nonempty bounded subsets of $\mathbb{R}$? Explain.

(2) Prove that $\sup(A \cup B) = \sup\{\sup A, \sup B\}$.

Cut this out and then put your answer here.

$\boxed{\textbf{27}}$. Variant of Exercise 2.3.11. §2.3 BS4p40

Let $\emptyset \neq A \subseteq B \subseteq \mathbb{R}$ and B be bounded above. Prove that

$$\inf B \leq \inf A \leq \sup A \leq \sup B.$$

Cut this out and then put your answer here.