▶ Put your Pin and Name into the LaTeX template (see first few lines) as well as on back of the last page in the upper right corner.
Turn in a stapled hard copy (copied one-sided) of your homework (folded to 4.25x11 size) at the beginning of class

Definition. Let $a \in \mathbb{R}$ and $\epsilon > 0$. Then the ϵ-neighborhood about a is the set

$$N_\epsilon(a) \;:=\; \{x \in \mathbb{R}\colon\; |x-a| < \epsilon\}\,.$$

Recall for two real numbers x and a, the distance $d(x,a)$ between x and a is

$$d(x,a) \;:=\; |x-a|$$

and for an $\epsilon > 0$

$$|x-a| < \epsilon \quad\Longleftrightarrow\quad -\epsilon < x-a < \epsilon \quad\Longleftrightarrow\quad a-\epsilon < x < a+\epsilon\,.$$

Thus we can think of an ϵ-neighborhood of a in several ways (see book's Figure 2.2.4 on page 35): Def2.2.7 BS4p35

$$\begin{aligned} N_\epsilon(a) &= \{x \in \mathbb{R}\colon d(x,a) < \epsilon\} \\ &= \{x \in \mathbb{R}\colon\; |x-a| < \epsilon\} \\ &= \{x \in \mathbb{R}\colon\; -\epsilon < x-a < \epsilon\} \\ &= \{x \in \mathbb{R}\colon a-\epsilon < x < a+\epsilon\} \\ &= (a-\epsilon, a+\epsilon) \quad \rightsquigarrow \text{ an open interval} \end{aligned}$$

An ϵ-neighborhood about a is also called an ϵ-neighborhood of a.
BTW: The notation $N_\epsilon(a)$ for a Neighborhood is more common than the book's notation of $V_\epsilon(a)$.

$\boxed{\textbf{20}}$. Variant of Exercise 2.1.17. (We will use Theorem 2.1.9 and Theorem 1 almost daily.) §2.1 BS4p31

On page 28, the textbook presents:

Theorem 2.1.9. If $a \in \mathbb{R}$ is such that $0 \le a < \epsilon$ for every $\epsilon > 0$, then $a = 0$.

Without using Theorem 2.1.9, prove the below Theorem 1.

Theorem 1. Let $x \in \mathbb{R}$ be such that for each $\epsilon > 0$

$$0 \le x \le \epsilon.$$

Then $x = 0$.

. . . .

$\boxed{\textbf{21}}$. Variant of Exercise 2.2.18a. §2.2 BS4p36

Let $x, y \in \mathbb{R}$. Let:

I be an interval with endpoint x and y

M be the midpoint of x and y

D be the distance between x and y.

Hints on (1a)/(1b). Draw yourself a picture. Stand on the the midpoint M. From M, how would you get to the upper endpoint of I? From M, how would you get to the lower endpoint of I?

(1a) Express $\max\{x, y\}$ as a function of M and D. Explain your solution geometrically (not algebraically).

$\langle$Your solution should look like $\max\{x, y\}$ = some mess/function involving M and D but not involving x nor y.$\rangle$

(1b) Express $\min\{x, y\}$ as a function of M and D. Explain your solution geometrically (not algebraically).

$\langle$Your solution should look like $\min\{x, y\} = =$ some mess/function involving M and D but not involving x nor y.$\rangle$

Hints on (2a)/(2b). Use your (1a)/(1b) along with the midpoint and distance formulas.

(2a) Express $\max\{x, y\}$ as a function of x and y. $\langle$Your solution should look like $\max\{x, y\}$ = some mess/function involving x and y but not involving M nor D.$\rangle$

(2b) Express $\min\{x, y\}$ as a function of x and y. $\langle$Your solution should look like $\min\{x, y\} = =$ some mess/function involving x and y but not involving M nor d.$\rangle$

. . . .

$\boxed{\mathbf{22}}$. Variant of Exercise 2.2.16. §2.2 BS4p36

Let $a \in \mathbb{R}$ and $\epsilon_1 > 0$ and $\epsilon_2 > 0$.

(1) Express $N_{\epsilon_1}(a) \cap N_{\epsilon_2}(a)$ as a δ-neighborhood about a.

So find $\delta > 0$ so that $N_{\epsilon_1}(a) \cap N_{\epsilon_2}(a) = N_\delta(a)$. Explain geometrically (not algebraically) why your δ works.

(2) Express $N_{\epsilon_1}(a) \cup N_{\epsilon_2}(a)$ as a γ-neighborhood about a.

So find $\gamma > 0$ so that $N_{\epsilon_1}(a) \cup N_{\epsilon_2}(a) = N_\gamma(a)$. Explain geometrically (not algebraically) why your γ works.

. . . .

$\boxed{\mathbf{23}}$. Variant of Exercise 2.2.17. §2.2 BS4p36

Let $a_1, a_2 \in \mathbb{R}$ such that $a_1 \neq a_2$.

(1) Find an $\epsilon > 0$ such that $N_\epsilon(a_1)$ and $N_\epsilon(a_2)$ are disjoint. Explain geometrically (not algebraically) why your ϵ works.

(2) Finish the statement of Lemma 1 below by giving an upper bound for ϵ (natually, ϵ is strictly positive).

Your solution should be of the form $0 < \epsilon <$(somthing) or $0 < \epsilon \leq$ (somthing). $\langle$Justification not needed.$\rangle$

Lemma 1 Let $a_1, a_2 \in \mathbb{R}$ such that $a_1 \neq a_2$. Then $N_\epsilon(a_1) \cap N_\epsilon(a_2) = \emptyset$ if and only if $0 < \epsilon \ldots$.

. . . .