► Put your Pin and Name into the LaTeX template (see first few lines) as well as on back of the last page in the upper right corner.
Turn in a stapled hard copy (copied one-sided) of your homework (folded to 4.25x11 size) at the beginning of class

You may use, without proving, previous homework problems as well as the below Thm, which we proved in class.

Theorem CUCC. A countable union of countable sets is a countable set. I.e., Thm1.3.12 BS4p20

If an index set I is countable and $\{A_i\}_{i\in I}$ is a collection of subsets of a universal set U,

then $\bigcup_{i\in I} A_i$ is a countable subset of U.

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17. Variant of Exercise 1.3.6. §1.3 BS4p22

Exhibit a bijection between $\mathbb{N}$ and a proper subset P of $\mathbb{N}$.

Remark. A set A is a proper subset of a set B means $A \subseteq B$ but $A \neq B$ (so $A \subsetneq B$).

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18. Variant of Exercise 1.3.8. §1.3 BS4p22

Conjecture 1. A countable union of finite subsets of $\mathbb{N}$ is a finite set.

Is the Conjecture 1 true or false? Justify your answer.

Instructions. You should understand (and this will not be repeated) the instructions means to do one of the following.

- If Conjecture 1 is false, then say Conjecture 1 is false and also justify by providing a counterexample (i.e., an example which shows the conjecture is false). Explain why your counterexample shows the conjecture is false.
- If Conjecture 1 is true, then say Conjecture 1 is true and also justify by providing a proof.

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19. Variant of Exercise 1.3.13. §1.3 BS4p22

Let $\mathcal{F}(\mathbb{N})$ be the collection of all *finite* subsets of $\mathbb{N}$.

► Since $\mathcal{F}(\mathbb{N})$ is a fingerful to type, let's make a newcommand (i.e., alias) for $\mathcal{F}(\mathbb{N})$. How about just fn?

On Overleaf, double click through here to learn this time-saving LaTex feature of newcommand's.

Note the syntax is \newcommand{\MyNewNameForIt}{TheLatexForIt}.

(1). Prove that the collection $\mathcal{F}(\mathbb{N})$ of all *finite* subsets of $\mathbb{N}$ is countable.

(2). Prove that $\mathcal{F}(\mathbb{N})$ is countable infinite.

HINT: BWOC, assume that $\mathcal{F}(\mathbb{N})$ is finite. Note $\mathcal{F}(\mathbb{N})$ is nonempty since $\{17\} \in \mathcal{F}(\mathbb{N})$. Since $\mathcal{F}(\mathbb{N})$ is nonempty and finite, $\mathcal{F}(\mathbb{N})$ has n elements for some $n \in \mathbb{N}$. So there is a bijection

$$f\colon \mathbb{N}^{\leq n} \to \mathcal{F}(\mathbb{N}).$$

⟨Now find a contradiction.⟩
