Problem 0. Read the handouts: List of Writing Guidelines and Writing Guidelines.
You can think of these guidelines, which you must follow, as a rubic for your proof writing.
HW 1's LaTex file should be helpful with LaTex. Recall the course homepage has LaTex help.
For HW problems, you may use, without proving, any previous HW problem (or problem part).
In proofs on homework (and exams), you may use LTGBG ⟨for *Let the givens be given*, as we do in class⟩.

Problem 4. Variant of Exercise 1.1.3. §1.1 BS4p10

Let S and T be sets of a universal set X.

(1) Prove that $S \subseteq T$ if and only if $S \cap T = S$.

(2) Prove that $S \subseteq T$ if and only if $S \cap T^C = \emptyset$. ⟨Here, $T^C = X \setminus T$.⟩

Proof. LTGBG. .... continue with your proof ... then end the proof with □

Problem 5. Variant of Exercise 1.1.7. §1.1 BS4p 11

Consider the subsets of $\mathbb{R}$ defined by, for each natural number n,

$$A_n = \{k \in \mathbb{N} \colon k \geq n\} .$$

Find the followings sets. No justification needed. Just type solutions directing into the statement of problem.

HINT: Draw a pictorial representation for A_1, A_2, A_3, A_{17} and then for an arbitrary A_n.

HINT: Some LaTex help; your solutions can look like:

$$\emptyset \ , \ \mathbb{N} \ , \ \mathbb{R} \ , \ \{n \in \mathbb{R} \colon n \leq 17\} \ , \ \{1, 2, 3, \ldots, 17\} \ .$$

(1).

$$\bigcap_{i=1}^{5} A_i = \text{cut this out and put your solution to (1) here}$$

(2).

$$\mathbb{N} \setminus \left(\bigcap_{i=1}^{5} A_i \right) =$$

(3).

$$\bigcup_{i=1}^{5} A_i =$$

(4).

$$\bigcap_{i \in \mathbb{N}} A_i =$$

(5).

$$\bigcup_{i \in \mathbb{N}} A_i =$$

Problem 6. Variant of Exercise 1.1.7. §1.1 BS4p 11

As in class, let $\mathbb{R}^{>0}$ be the strictly positve real numbers $\langle$so $\mathbb{R}^{>0} = \{x \in \mathbb{R} \colon x > 0\}\rangle$.

For each $r \in \mathbb{R}^{>0}$, define T_r to be the closed interval $[-r^2, r^2]$ of $\mathbb{R}$, i.e.,

$$T_r = \left\{x \in \mathbb{R} \colon -r^2 \le x \le r^2\right\}. \tag{1}$$

Find the following sets. No justification needed.

HINT: Draw a pictorial representation for $T_{\frac{1}{3}}$, $T_{\frac{1}{2}}$, T_1, T_2, T_3, and then for an arbitrary T_r.

REMARK: Just type solutions directing into the statement of problem.

Let the index set be $I = \{m \in \mathbb{N} \colon 1 \le m \le 6\}$.

(0). $\langle$sample done for you$\rangle$

$$\bigcap_{i \in I} T_i = [-1, 1]$$

(1).

$$\bigcup_{i \in I} T_i =$$

The index set is $\mathbb{N}$

(2).

$$\bigcap_{i \in \mathbb{N}} T_i =$$

(3).

$$\bigcup_{i \in \mathbb{N}} T_i =$$

The index set is $\mathbb{R}^{>0}$

(4).

$$\bigcap_{i \in \mathbb{R}^{>0}} T_i =$$

(5).

$$\bigcup_{i \in \mathbb{R}^{>0}} T_i =$$

Problem 7. Variant of Exercise 1.1.21. §1.1 BS4p11

Let $f \colon X \to Y$ and $g \colon Y \to Z$ be bijective functions.

Prove the composite function $g \circ f$ is bijective.

HINT: Note $g \circ f \colon X \to Z$; think of as $(g \circ f) \colon X \xrightarrow{f} Y \xrightarrow{g} Z$.

Problem 8. Variant of Exercise 1.1.22. §1.1 BS4p11

Let $f \colon X \to Y$ and $g \colon Y \to Z$ be functions such that the composite function $g \circ f$ is injective.

(1) Prove that f is injective.

(2) Need g be injective? Either proof or give a counterxample. ⟨See below remark.⟩

REMARK: If g need not be injective, then give an example of an $f \colon X \to Y$ and $g \colon Y \to Z$ such that $g \circ f$ is injective but g is not injective. Similar to the way we thought of functions in the class lecture on the *Function and Sets* handout, try taking X, Y, and Z to be finite sets (e.g., $X = \{x_1, x_2\}$ or $X = \{x_1, x_2, x_3\}$) and say how the functions f and g are defined (e.g., $f(x_1) = y_1$).

Problem 9. Variant of Exercise 1.1.22. §1.1 BS4p11

Let $f \colon X \to Y$ and $g \colon Y \to Z$ be functions such that the composite function $g \circ f$ is surjective.

(1) Prove that g is surjective.

(2) Need f be surjective? Either proof or give a counterxample. ⟨See below remark.⟩

REMARK: If f need not be surjective, then give an example of an $f \colon X \to Y$ and $g \colon Y \to Z$ such that $g \circ f$ is surjective but f is not surjective. Similar to the way we thought of functions in the class lecture on the *Function and Sets* handout, try taking X, Y, and Z to be finite sets (e.g., $X = \{x_1, x_2\}$ or $X = \{x_1, x_2, x_3\}$) and say how the functions f and g are defined (e.g., $f(x_1) = y_1$).

Problem 10. Variant of Exercise 1.1.23. §1.1 BS4p11

Let $f \colon X \to Y$ and $g \colon Y \to Z$ be functions. Let C be a subset of Z. Prove that

$$(g \circ f)^{-1}[C] = f^{-1}\left[g^{-1}[C]\right].$$

HINT: By definition of the preimage of the set C under the function $g \circ f$, what does it mean for $z \in (g \circ f)^{-1}[C]$? By definition of the preimage of the set $g^{-1}[C]$ under the function f, what does it mean for $z \in f^{-1}[g^{-1}[C]]$?