Problem 0. Read the handouts

- List of Writing Guidelines,
- Writing Guidelines.

You can think of these guidelines, which you must follow, as a rubic for your proof writing.

Section 1.2 covers the Principle of Math Induction (PMI). Since induction is taught in Math 300 (a prerequiste for this course) we will not cover §1.2 in class. Induction is summarized in the handout

- Induction (including further writing guidelines for PMI).

After reviewing the above handouts, do the below problems.

Recall the course homepage has LaTex help, e.g. Getting Started.

Problem 1. Variant of Exercise 4.1.19 from Sundstrom. §1.2

Conjecture 1. For each $n \in \mathbb{N}$

$$\sum_{j=1}^{n} j = \frac{n^2+n+1}{2}. \tag{1}$$

Note that when $n = 1$

$$\sum_{j=1}^{n} j = \sum_{j=1}^{1} j = 1 \qquad \text{but} \qquad \frac{n^2+n+1}{2} = \frac{1^2+1+1}{2} = \frac{3}{2}. \tag{2}$$

Thus Conjecture 1 is false; indeed, the calculations in (2) show the formula in (1) fails when $n = 1$.

Incorrect Proof of Conjecture 1. We shall show that

$$\sum_{j=1}^{n} j = \frac{n^2+n+1}{2} \tag{$P(n)$}$$

for each $n \in \mathbb{N}$ by induction.

For the inductive step, fix $n \in \mathbb{N}$. Assume the inductive hypothesis, which is

$$\sum_{j=1}^{n} j = \frac{n^2+n+1}{2}. \tag{IH}$$

We shall show the inductive conclustion, which is

$$\sum_{j=1}^{n+1} j = \frac{(n+1)^2+(n+1)+1}{2}. \tag{IC}$$

Towards showing the inductive conclusion (IC), note that

$$\sum_{j=1}^{n+1} j = \left[\sum_{j=1}^{n} j\right] + (n+1)$$

and by (IH) we get (note how this is LaTexed with intertext)

$$= \left[\frac{n^2+n+1}{2}\right] + (n+1)$$

and now by algebra

$$\begin{aligned}
&= \frac{(n^2+n+1)+2(n+1)}{2} \\
&= \frac{(n^2+n+1)+(2n+2)}{2} \\
&= \frac{n^2+3n+3}{2} \\
&= \frac{(n^2+2n+1)+(n+2)}{2} \\
&= \frac{(n^2+2n+1)+(n+1)+1}{2} \\
&= \frac{(n+1)^2+(n+1)+1}{2}.
\end{aligned}$$

We have just show that (IC) holds. This concluded the inductive step.

Thus, by induction, the formula in $(P(n))$ holds for each $n \in \mathbb{N}$. □

What is wrong with the *incorrect proof* of the false Conjecture 1? ⟨Your answer can be just a few sentences.⟩

Problem 2. Variant of Exercise 4.1.3c from the Sundstrom book. §1.2

Using math induction, prove that for each $n \in \mathbb{N}$

$$\sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2.$$

Be sure follow all Writing Guidelines (see Problem 0).

Proof. Cut these sentence out and put your proof here between the begin/end proof. Problem 1 should help you with LaTeX. Recall a nice Overleaf feature is that if you double click (with left mouse key) at a place in the PDF file, then Overleaf indicated to you the corresponging place in the TeX file. This feature makes it easy to compare the PDF output to the LaTex input. □

Problem 3. §1.2

Using Math Induction, prove that every natural number greater than 3 may be written as an integer linear combination of the numbers 2 and 5; that is, if $m \in \mathbb{N}^{\geq 4}$ then there exists $x, y \in \mathbb{Z}$ such that $m = 2x + 5y$. ⟨Hint. Why is strong induction helpful?⟩

Proof. Cut this sentence out and put your proof here between the begin/end proof. □