INSTRUCTIONS:
1. To receive credit you must:
   a. WORK IN A LOGICAL FASHION, SHOW ALL YOUR WORK, 
      INDICATE YOUR REASONING
   b. when applicable put your answer on/in the line/box provided
   c. if no such line/box is provided, then box your answer.
2. The MARK BOX indicates the problems along with their points.
   Check that your copy of the exam has all of the problems.
3. As indicated on the syllabus:
   a. allowed is a calculator
   b. allowed are the class handouts: table of integrals, calculus formula sheet, 
      and informal summary (along with your personal scribbles on them)
   c. not allowed are other notes and books.
4. This exam covers (from Vector Calculus by Marsden & Tromba, 4th ed.) :
   the whole thing .

WARNING:
Problems are NOT ordered by increasing level of difficulty.
Problems are ordered by the ordering in which we covered the topic in class.

The sources of the questions on this exam are:
(1) typical of § 1.3
(2) just me thinking
(3) from your homework
(4) from Edwards&Penny, 3rd ed., pg 885
(5) from your homework
(6) like problems from previous exams
1. **Planes**

**1a.** An equation, in the form of \( ax + by + cz = d \), of the plane that passes through the points \((2, 0, 0)\) and \((0, 3, 0)\) and \((1, 2, 3)\) is:

\[
\text{(Write the equation here)}
\]

**1b.** In the space below, clearly verify that the above 3 given points do indeed satisfy the equation that you wrote in 1a.
2. **Parameterization & Surface Area**

Let \( m, r_1 \) and \( r_2 \) be fixed arbitrary positive constants with \( r_1 < r_2 \).

Consider the frustum \( S \) that is the surface of the cone:

\[
z = m \sqrt{x^2 + y^2}
\]

that sits between the planes \( z = mr_1 \) and \( z = mr_2 \).

So \( S \) is just the side part and does **not** include the top nor bottom "lids".

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2a. Make a rough sketch of the \( S \), labeling the two points on \( S \) of the form \( (0, ?, mr_1) \) and \( (0, ?, mr_2) \), indicating what \( ? \) are.

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2b. A parameterization of \( S \) is:

\[
\Phi(r, \theta) = \text{______________________________},
\]

over the domain

\[
D = \{(r, \theta) : \quad \quad \quad \quad \quad \quad \quad \text{and} \quad \quad \quad \quad \quad \quad \quad \}
\]

continued
2c. A (not necessarily normalized) normal vector to $S$ is:
\[ \vec{T}_r \times \vec{T}_\theta = \text{______________________________} ; \]
and this vector has length : ____________________________.

2d. The surface area of $S$ can be expressed as the following REIMANN integral over $D$:
\[ \text{SA} = \text{______________________________} . \]

2e. Integrating the integral in 2d gives that the surface area of $S$ is:
\[ \text{SA} = \text{______________________________} . \]
2f. Recall that the surface area of the curved side part of a right circular cone, with base of radius $r$ and height $h$, is $\pi r \sqrt{r^2 + h^2}$. Why is this in agreement with your solution to 2e? Work Logically.
3. **Green’s Theorem**  Let $a$ and $b$ be fixed arbitrary positive constants. Consider the inside of an ellipse:

$$D = \left\{ (x, y) : \frac{x^2}{a^2} + \frac{y^2}{b^2} \leq 1 \right\}$$

along with its boundary:

$$\delta D = \left\{ (x, y) : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \right\}.$$

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3a. A parameterization of $\delta D$, oriented in the counterclockwise direction, is:

$$\vec{c}(t) = \text{__________________________}.$$  

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3b. Using Green’s Theorem, the area of $D$ can be expressed as the following **LINE** integral:

$$A = \text{__________________________}.$$  

---

3c. The line integral in 3b can be expressed as the following **REIMANN** integral:

$$A = \text{__________________________}.$$  

---

3d. Integrating the integral in 3c gives that the area of $D$ is:

$$A = \text{__________________________}.$$
4. **Stokes' Theorem**

Let \( C \) be the ellipse in which the plane

\[
z = y + 3
\]

intersects the cylinder

\[
x^2 + y^2 = 1,
\]

oriented in the counterclockwise direction. Let

\[
\vec{F}(x, y, z) = (3z, 5x, -2y).
\]

Then:

\[
\oint_C \vec{F} \cdot d\vec{s} = \text{______________________________}.
\]

A number should go on the above line; clearly indicate your reasoning below.
5. **Gauss’ Theorem**

Let

\[ \vec{F}(x, y, z) = \langle x, y, 3 \rangle \]

and \( S \) be the surface of the unit sphere \( x^2 + y^2 + z^2 = 1 \), with the outward orientation.

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5a. The flux of \( \vec{F} \) across \( S \) can be expressed as the following SURFACE integral over \( S \):

\[
\text{flux} = \quad \text{(expression for the surface integral)}
\]

5b. The flux is:

\[
\text{flux} = \quad \text{(number)}
\]

A number should go on the above line; clearly indicate your reasoning below.
6. **Conservative Fields**

A puffo is running along a path \( C \) given by

\[
\vec{c}(t) = \langle (2 - t)e^t, t, 2t \rangle ,
\]

for \( 0 \leq t \leq 2 \), while being subjected to a force field

\[
\vec{G}(x, y, z) = \langle (1 + x)e^{x+y}, xe^{x+y} +, 5y \rangle.
\]

---

6a. The work done by \( \vec{G} \) on the puffo as he moves along the path \( \vec{c} \) can be expressed by the following line integral:

\[
W = \int_C \vec{G} \cdot d\vec{r}.
\]

6b. The work done by \( \vec{G} \) on the puffo as he moves along the path \( \vec{c} \) is:

\[
W = \int_C \vec{G} \cdot d\vec{r}.
\]

A number should go on the above line; clearly indicate your reasoning below. Some cleverness is needed. Is \( \vec{G} \) conservative? If not, can you find a conservative field \( \vec{F} \) that is similar to \( \vec{G} \) and then take advantage of the similarity?