▷. Below are Writing Guidelines from the commly used textbook *Book of Proof* by Richard Hammack. The above link is to Edition 3.3 and the below is taken from Section 5.3 (p. 133–135).

This book has some examples of bad usage (marked with ×) and good usage (marked with ✓).

Note how easy it is to adjust a bad × to a good *checked*.

5.3 Mathematical Writing

Now that we have begun writing proofs, it is a good time to contemplate the craft of writing. Unlike logic and mathematics, where there is a clear-cut distinction between what is right or wrong, the difference between good and bad writing is sometimes a matter of opinion. But there are some standard guidelines that will make your writing clearer. Some are listed below.

1. **Begin each sentence with a word, not a mathematical symbol.** The reason is that sentences begin with capital letters, but mathematical symbols are case sensitive. Because x and X can have entirely different meanings, putting such symbols at the beginning of a sentence can lead to ambiguity. Here are some examples of bad usage (marked with ×) and good usage (marked with ✓):

   A is a subset of B. ×
   The set A is a subset of B. ✓

   x is an integer, so $2x+5$ is an integer. ×
   Because x is an integer, $2x+5$ is an integer. ✓

   $x^2-x+2=0$ has two solutions. ×
   $X^2-x+2=0$ has two solutions. × (and silly too)
   The equation $x^2-x+2=0$ has two solutions. ✓

2. **End each sentence with a period,** even when the sentence ends with a mathematical symbol or expression.

   Euler proved that $\sum_{k=1}^{\infty} \frac{1}{k^s} = \prod_{p\in P} \frac{1}{1-\frac{1}{p^s}}$ ×

   Euler proved that $\sum_{k=1}^{\infty} \frac{1}{k^s} = \prod_{p\in P} \frac{1}{1-\frac{1}{p^s}}$. ✓

   Mathematical statements (equations, etc.) are like English phrases that happen to contain special symbols, so use normal punctuation.

3. **Separate mathematical symbols and expressions with words.** Not doing this can cause confusion by making distinct expressions appear to merge. Compare the clarity of the following examples.

   Because $x^2 - 1 = 0$, $x = 1$ or $x = -1$. ×
   Because $x^2 - 1 = 0$, it follows that $x = 1$ or $x = -1$. ✓

   Unlike $A \cup B$, $A \cap B$ equals $\emptyset$. ×
   Unlike $A \cup B$, the set $A \cap B$ equals $\emptyset$. ✓

4. **Avoid misuse of symbols.** Symbols such as $=$, $\leq$, $\subseteq$, $\in$, etc., are not words. While it is appropriate to use them in mathematical expressions, they are out of place in other contexts.

   Since the two sets are $=$, one is a subset of the other. ×
   Since the two sets are equal, one is a subset of the other. ✓

   The empty set is a $\subseteq$ of every set. ×
   The empty set is a subset of every set. ✓

   Since a is odd and x odd $\Rightarrow x^2$ odd, a^2 is odd. ×
   Since a is odd and any odd number squared is odd, a^2 is odd. ✓

5. **Avoid using unnecessary symbols.** Mathematics is confusing enough without them. Don't muddy the water even more.

   No set X has negative cardinality. ×
   No set has negative cardinality. ✓

6. **Use the first person plural.** In mathematical writing, it is common to use the words "we" and "us" rather than "I," "you" or "me." It is as if the reader and writer are having a conversation, with the writer guiding the reader through the details of the proof.

7. **Use the active voice.** This is just a suggestion, but the active voice makes your writing more lively. (And briefer too.)

   The value $x = 3$ is obtained through division of both sides by 5. ×
   Dividing both sides by 5, we get $x = 3$. ✓

8. **Explain each new symbol.** In writing a proof, you must explain the meaning of every new symbol you introduce. Failure to do this can lead to ambiguity, misunderstanding and mistakes. For example, consider the following two possibilities for a sentence in a proof, where a and b have been introduced on a previous line.

Since $a \mid b$, it follows that $b = ac$. ×
Since $a \mid b$, it follows that $b = ac$ for some integer c. ✓

If you use the first form, then the reader may momentarily scan backwards looking for where the c entered into the picture, not realizing at first that it came from the definition of divides.

9. **Watch out for "it."** The pronoun "it" causes confusion when it is unclear what it refers to. If there is any possibility of confusion, you should avoid "it." Here is an example:

   Since $X \subseteq Y$, and $0 < |X|$, we see that it is not empty. ×

   Is "it" X or Y? Either one would make sense, but which do we mean?

   Since $X \subseteq Y$, and $0 < |X|$, we see that Y is not empty. ✓

10. **Since, because, as, for, so.** In proofs, it is common to use these words as conjunctions joining two statements, and meaning that one statement is true and as a consequence the other true. The following statements all mean that P is true (or assumed to be true) and as a consequence Q is true also.

Q since P	Q because P	Q, as P	Q, for P	P, so Q
Since P, Q	Because P, Q	As P, Q		

    Notice that the meaning of these constructions is different from that of *"If P, then Q,"* for they are asserting not only that P implies Q, but **also** that P is true. Exercise care in using them. It must be the case that P and Q are both statements **and** that Q really does follow from P.

    $x \in \mathbb{N}$, so $\mathbb{Z}$ ×
    $x \in \mathbb{N}$, so $x \in \mathbb{Z}$ ✓

11. **Thus, hence, therefore, consequently.** These adverbs precede a statement that follows logically from previous sentences or clauses. Be sure that a statement follows them.

    Therefore $2k+1$. ×
    Therefore $a = 2k+1$. ✓

12. **Clarity is the gold standard of mathematical writing.** If you think breaking a rule makes your writing clearer, then break the rule.

Your mathematical writing will evolve with practice. One of the best ways to develop a good mathematical writing style is to read other people's proofs. Adopt what works and avoid what doesn't.