▷. Our Writing Guidelines unify the guidelines from the two commonly used Math 300 textbooks:

[S] Mathematical Reasoning. Writing and Proof by Ted Sundstrom, Version 3.
(Appendix A on p. 492-496, also, the guidelines are introduced within sections: 1.1, 1.2, 3.1, 3,3, 4.1.)

[H] *Book of Proof* by Richard Hammack (Edition 3.3, §5.3, p. 133–135)
This book has nice examples of bad usage (marked with ×) and good usage (marked with ✓).

Often verbiage is included not in [H] nor [S]. Sometimes verbiage is taken directly from [H] and [R]. Often a reference is given (i.e., [S2] refers to the second guideline in [S]). If you want further information, read the given reference. The right margin indicated the section in [S] the WG was introduced.

Your math writing will evolve with practice. One of the best ways to develop a good mathematical writing style is to read other people's proofs. Adopt what works and avoid what doesn't.

WG1–WG16

WG1. Know your audience. §3.1
Our intended audience is a *confused* fellow classmate in this course. So if in doubt on whether or not to include some verbiage, then ask yourself: "would this verbiage help my confused friend?".
For further information see [S1].

WG2. Use proper grammar. §3.1
Use complete sentences. Avoid run-on sentences. Start sentences with a capital letter. Do not forgot punctuation. Start a new paragraph by indenting (in LaTex, just leave a blank line).
For further information see [S7] and [H2].

WG3. Keep it simple. §3.1
Use simple, declarative sentences and short paragraphs, each with a simple point. Order your argument directly (following the implication arrows in your thinking land), which may differ from the order you did your thinking land in; i.e., don't drag your reader through the mud. Do not write out definitions in the book (the reader knows the definitions). Do not include facts not used in your proof.
For further information see [S14].

WG4. Watch out for (potentially unclear) words such as: it, this, that.
The use of such words cause confusion when it is unclear to what the word is referring. If there is any possibility of confusion, you should avoid the word. For example, instead of writing "If $X \subsetneq Y$ then it has strictly more elements." write "If $X \subsetneq Y$ then Y has strictly more elements than X."
For further information see [H9].

WG5. Display (by centering) important equations and mathematical expressions. §1.2
Equations and manipulations are often an integral part of a proof. Do not write equations, algebraic manipulations, or formulas in one column with reasons given in another column (as is often done in geometry texts). Do not write down an equation that you need to show (even with a question mark above the equal sign) as if it is true before you have shown it is true. Important equations and manipulations should be displayed (i.e., centered) and if one side of an equation does not change, it should not be repeated.
Example.

Using algebra, we obtain

$$\begin{aligned} x \cdot y &= (2m+1)(2n+1) \\ &= 4mn + 2m + 2n + 1 \\ &= 2(2mn + m + n) + 1. \end{aligned}$$

Since m and n are integers, we conclude that . . . .
For further information see [S9].

WG6. Equation numbering guidelines. §1.2
If it is necessary to refer to an equation later in a proof, then that equation should be centered

and displayed, and it should be given a number. The number for the equation should be written in parentheses on the same line as the equation at the right-hand margin.

Example.
Since x is an odd integer, there exists an integer n such that

$$x = 2n + 1. \tag{1}$$

Later in the proof, there may be a line such as: Then, using the result in (1), we obtain . . . . Please note that we should only number those equations we will be referring to later in the proof.
For further information see [S10].

WG7. Use English and minimize the use of cumbersome and unnecessary notation. §3.1
In a sentence, do not use: ⟨will learn these symbols in Math 300⟩

$$\forall \ , \ \exists \ , \ \therefore \ . \qquad \text{(do not use)}$$

In a sentence, do not use logical connectives, such as: ⟨will learn these symbols in Math 300⟩

$$\wedge \ , \ \vee \ , \ \implies \ , \ \Leftrightarrow \ , \ \sim \ , \ \neg \ . \qquad \text{(do not use)}$$

Within a mathematical expression, you may use the standard math notations such as:
⟨knew these symbols before Math 300⟩

$$= \ , \ < \ , \ \leq \ , \ + \ , \ - \ , \ \subseteq \ , \ \sum_{i=1}^{n} \ . \qquad \text{(can use)}$$

You can use symbols as in (can use) provided the symbols are in a math expression, e.g.," We know $1 + 1 = 2$.". However, the symbol cannot replace the word. E.g., do not say "The numbers x and y are =." but rather say "The numbers x and y are equal." (or maybe "So we get $x = y$.").
For further information see [S12], [H4], [H5].

WG8. Separate mathematical symbols/expressions with English words. §3.1
The reason is clarity. For example, the sentence "Since $x > 0$, $x^2 > 0$" is hard to read but by just adding a *filler words* such as "Since $x > 0$, we have $x^2 > 0$" makes the sentence easier on the eye. Basically, avoid putting a punctuation symbol between 2 math symbols. Having some go-to *filler words* handy is helpful.
For further information see [H3] and [S11].

WG9. Begin each sentence with a word, not a math symbol. §3.1
Sentences begin with capital letters, but math symbols are case sensitive. Because n and N can have entirely different meanings in math, putting such symbols at the beginning of a sentence can lead to ambiguity.
For further information see [H1], which as some nice examples, and [S11].

WG10. Do not use $*$ for multiplication or $\hat{}$ for exponents. For complicated fractions, do not use /. §3.1
Leave computer science notation for writing computer code. The use of CS notation makes it difficult for humans to read. In addition, avoid using / for division for a complicated fraction. For example, it is very difficult to read $(x^3 - 3x^2 + 1/2)/(2x/3 - 7)$; the fraction

$$\frac{x^3 - 3x^2 + \frac{1}{2}}{\frac{2x}{3} - 7}$$

is much easier to read.
For further information see [S6].

WG11. Use italics for variables when using a word processor. §1.2
Of course, most of us cannot italicize when handwriting so just do the best you can. If you are using, e.g., the work processor LaTex, just be sure you are in math mode when typing a variable (e.g., between dollar signs or is a displayed math environment, such as *equation* or *align*).
For further information see [S5].

WG12. **Use the pronouns "we" rather than "I", "you", or "me".** §1.2
The usual convention in math is to use "we" since it is then as if the reader and writer are having a conversation, with the writer guiding the reader through the details of the proof.
For further information see [H6] and [S4].

WG13. **Write a first draft of your proof and then revise it.** §3.1
Remember that a proof should be written so a reader can easily read and understand the reasoning in the proof. Be clear and concise. Write your proof in the order of your thinking land (i.e., scratch work) implication arrows rather than the order you might have figured out the proof; don't pull the reader through the mud. Include needed details but do not ramble by including unused information. Do not be satisfied with the first draft of a proof. Read it over and refine it.
For further information see [S15].

WG14. **Keep your reader informed.** §3.3
In the beginning of your proof, if you do not indicate your proof method then the reader will think §4.1
you are using a direct proof. Thus if you are not using a direct proof, then in the beginning of §1.2
your proof, inform your reader which proof method you are using.

Make clear and precise what your hypotheses/assumptions are. Justify each claim you make (e.g.: *So xy is odd by definition of odd* or *So xy is odd by Lemma POO).*

Indicate when the proof is completed. You can put the "end of proof symbol" □ (or ■) at the right-hand margin of the last line of your proof. Another way is to have the one sentence closing paragraph *This completes the proof.*
For further information see [S8], [S13].

WG15. **Format of Direct Proof of a conditional statement** (i.e., of P (the hypothesis) $\implies$ Q (the conclusion)).
Start the first line with "Proof." Begin the 1st (opening) paragraph of the proof on the same line. Then clearly state your hypothesis (or hypotheses, the plural). When stating your hypothesis, you should (natually) be setting your notation. Then clearly state what you want to show (WTS) or what you want to find (i.e., the conclusion).

Next move onto the middle paragraph(s) of your proof, where you argue why your hypothesis implies your WTS or finds what you want to find. This is the *meat* of your proof. Remember to justify each assertion that is made. Write this part in the order of your thinking land implication arrows rather than the order you might have figured out the proof; don't pull the reader through the mud. Include needed details but do not ramble by including unused information.

Once you have finished arguing that the hypothesis implies the conclusion, inform your reader your proof is finished (with the □ or the sentence *This completes the proof.*).

As an example, consider Thm. 1.

Theorem 1. The sum of two even integers is an even integer.

The (direct) proof of Theorem 1 could take the below form.

Proof. Let x and y be an even integers. We will show that $x + y$ is an even integer.

The next paragraph(s) is the "meat" of the proof where, using your hypothesis that x and y are even integers, you agrue that $x + y$ is an even integer. □

For further information see [S2], [S3], [S8], [S13].

WG16. **Format of Other Proofs.**
If not proving a conditional statement by a direct proof, then indicate the method of proof at the beginning of your proof. E.g., if you are using a proof by contrapositive, your proof's first line can be:
Proof. We shall use a proof by contrapositive.

The remainder of the proof varies by method but follows basically the same guidelines set forth in the above **Format of Direct Proof** and **Keep your reader informed**.
For further information see [S8].