1.5.1

*Proof by Contraposition of $P \rightarrow Q$

Assume $\neg Q$

Therefore, $\neg P$

Thus, $\neg Q \rightarrow \neg P$

Therefore, $P \rightarrow Q$

*For the conditional sentence $P \rightarrow Q$, we make use of tautology $(P \rightarrow Q) \iff (\neg Q \rightarrow \neg P)$.

Since $P \rightarrow Q = \neg Q \rightarrow \neg P$, we first give proof of $\neg Q \rightarrow \neg P$ and then conclude by replacement that $P \rightarrow Q$

1.5.2

*Proof by Contradiction

Suppose $\neg P$

Therefore, $Q$

Therefore, $\neg Q$

Hence, $Q \land \neg Q$ a contradiction

Thus, $P$

*Logic is that if a statement cannot be false, then it must be true. This proof is based on the tautology $P \iff [(\neg P) \rightarrow (Q \land \neg Q)]$. This method can be applied to any proposition, $P$, but may require some insight to determine a useful $Q$. 
1.5.3

* Two part Proof of $P \iff Q$

(i) Show $P \implies Q$

(ii) Show $Q \implies P$

Therefore, $P \iff Q$

*Proof of biconditional sentences $P \iff Q$ often make use of the tautology $(P \iff Q) \iff (P \implies Q) \land (Q \implies P)$

* Biconditional Proof of $P \iff Q$

$P \iff R_1 \iff R_2 \iff R_n \iff Q$.

* Proof by Contradiction

Suppose $\neg P$

Therefore $\neg Q$

Hence $Q \implies Q$, a contradiction

Thus $P$

* Logic is that if a statement cannot be false, then it must be true. The proof is based on the tautology $P \iff (P \iff Q)$.

This method can be applied to any proposition $P$, but may require some insight to determine a useful $Q$. 