1.5 Informal Summary

Basic Proof Methods

1.5.1 Definition. An **Indirect Proof** is a proof that uses techniques based on tautologies that replace the statement to be proved by an equivalent statement or statements.

1.5.2 Definition. A **Contrapositive Proof** for a conditional sentence \( P \rightarrow Q \) makes use of the tautology \( (P \rightarrow Q) \equiv (\sim Q \rightarrow \sim P) \).

1.5.3 Note. Proof by Contraposition of \( P \rightarrow Q \)

**Proof.** Assume \( \sim Q \).

\[ 
\begin{align*}
\text{Therefore, } & \sim P. \\
\text{Thus, } & \sim Q \rightarrow \sim P. \\
\text{Therefore, } & P \rightarrow Q.
\end{align*}
\]

1.5.4 Definition. **Proof by Contradiction** is a technique based on the statement that "if a statement cannot be false, then it must be true."

1.5.5 Note. Proof of \( P \) by Contradiction

**Proof.** Suppose \( \sim P \).

\[ 
\begin{align*}
\text{Therefore, } & Q. \\
\text{Therefore, } & \sim Q. \\
\text{Hence, } & Q \wedge \sim Q \text{ a contradiction.} \\
\text{Thus, } & P.
\end{align*}
\]

1.5.6 Note. **Two-Part Proof of** \( P \rightarrow Q \)

**Proof.**

i) Show \( P \rightarrow Q \).

ii) Show \( Q \rightarrow P \).

Therefore, \( P \rightarrow Q \).
1.5.7 Definition: The Parity of an integer is the attribute of being either even or odd.

1.5.8 Note: Biconditional proof of $P \iff Q$

\[
\begin{align*}
& \text{Proof:} \\
& P \iff R_1 \\
& \iff R_2 \\
& \ldots \\
& \iff R_n \\
& \iff Q
\end{align*}
\]

Where $R_1, R_2, \ldots, R_n$ are a sequence of equivalent statements.

1.5.9 Definition: Consistent Axiom Systems are systems of structures satisfying all the axioms.

1.5.10 Definition: An Undecidable statement is one which neither it or its negation can be proved true. Their truth is independent of the truth of the axioms.