A Transition to Advanced Mathematics by Smith, et. Al (7th edition)

Definitions:

- **Indirect proofs**: the techniques that are based on tautologies that replace the statement to be proved by an equivalent statement of statements.
- **Example of a contrapositive proof**
  - (P \implies Q) \iff (\neg Q \implies \neg P)
  - Since both of those are equivalent statements, we first give a proof of \neg Q \implies \neg P and then conclude by replacing that with P \implies Q.
    - PROOF OF CONTRAPOSITION OF P \implies Q
      Assume \neg Q
      Therefore, \neg P
      Thus, \neg Q \implies \neg P
      Therefore, P \implies Q

- **Proof by Contradiction**
  - A proof that if a statement cannot be false, then it must be true.
    - PROOF BY CONTRACTION
      Suppose \neg P.
      Therefore, Q
      Therefore, \neg Q
      Hence, Q \land \neg Q a contradiction
      Thus, P.

Proofs by contradiction can be applied to any proposition P, whereas direct proofs and proofs by contraposition can be used only for conditional sentences.

- **Parity**: the attribute of being either odd or even
- **Consistent axiom systems**: statements that neither the negation nor the statement can be proved.
- **Undecidable**: truth is independent of the truth of the axioms.

- EXAMPLES (in book) p .40
  - Let m be an integer. Prove that if \( m^2 \) is even, then m is even.

  \( \{ P="m^2 \text{ is even"} \ \ Q= "m \text{ is even"} \} \)

  Proof: suppose that the integer m is not even. (\neg Q) then m is odd so \( m=2k+1 \) for some integer k. then:

  \[ m^2 = (2k+1)^2 = 4k^2 + 4k + 1 = 2(2k^2) \]
Since $m^2$ is twice an integer, plus 1, $m^2$ is odd. Therefore, $m^2$ is not even.

Thus, if $m$ is not even, then $m^2$ is not even. By contraposition, if $m^2$ is even, then $m$ is even.