**Unformal Summaries** 1.4

**Theorem** - a statement that describes a pattern or relationship among quantities or structure.

**Proof** - a justification of the truth of a theorem.

**Axioms** (Postulates) - an initial set of statements.

**Undefined terms** - concepts fundamental to the context of study.

**Definitions** - new concepts.

*Replacement Rule* - often used in combination with the equivalences of.

Modus Ponens - the most fundamental rule of reasoning, which is based on the tautology:

\[ P \land (P \Rightarrow Q) \Rightarrow Q. \]
modus ponens rule - states that "At any time after $P$ and $P \implies Q$ appear in a proof, state that $Q$ is true."

*direct proof* - the first and most important proof method of the statement of the form $P \implies Q$, which proceeds in a step by step fashion from the antecedent $P$ to the consequent $Q$.

Replacement Rule Example:

Suppose we have been able to establish the step:

"It is not the case that"

$x$ is even and prime.

Because the form of this statement is $\neg(P \land Q)$, where $P$ is "$x$ is even" and $Q$ is "$x$ is prime."

We may deduce that:

"$x$ is not even or"

$x$ is not prime.

- which has form $\neg P \lor \neg Q$.

We have applied the replacement rule; using one of De Morgan's laws.
A working knowledge of equivalences of Theorems 1.1.1 and 1.2.2 is essential.

The Replacement Rule allows you to use definitions in two ways:

1) If you are told or have shown that \( x \) is odd, then you can correctly state that for some natural number \( k \), \( x = 2k + 1 \).

   - You now have an equation to use.

2) If you need to prove that \( x \) is odd, then the definition gives you something equivalent to work toward:

   "It suffices to show that \( x \) can be expressed as \( x = 2k + 1 \) for some \( k \)."

   - These two ways are useful in writing proofs.

Tautology Rule Example:

If a proof involves a real number \( x \), you may at anytime assert "Either \( x > 0 \) or \( x \leq 0 \)" since this is an instance of the tautology \( P \lor \neg P \).