Def. Theorem - a statement that describes a pattern or relationship among quantities or structures.

Proof - a justification of the truth

Axioms (or postulates) - an initial set of statements that are assumed to be true.

Undefined terms - concepts fundamental to the context of study.

Replacement rule - used in a combination with the true equivalences of Theorems 1.1.1 and 1.2.2 to rewrite a statement involving logical connectives.

Tautology rule - at anytime state a sentence whose symbolic translation is a tautology.

Modus Ponens - most fundamental rule of reasoning:
- At anytime after P and P → Q appear in a proof, state that Q is true.

Direct proof - (first and most important proof method) - a statement of the form P → Q, which proceeds in a step-by-step fashion from the antecedent P to the consequent Q. Since P → Q is false only when P is true and Q is false, it suffices to show that this cannot happen.
You may

→ At any time state an assumption, an axiom, or a previously proved result.

→ At any time state a sentence equivalent to any statement in the proof.

→ At any time state a sentence whose symbolic translation is a tautology.

→ At any time after P and P → Q appear in a proof, state that Q is true.
  *(This goes with Modus Ponens Rule)*

Keep in mind

→ We can only be certain that what we proved will be true when all the assumptions are true.

→ Modus Ponens tautology is based on \[ P \land (P \rightarrow Q) \rightarrow Q \]

→ Every step within a proof must be justifiable.

→ Direct Proof of \( P \rightarrow Q \)

Assume \( P \)

\[ \therefore Q. \]

Thus, \( P \rightarrow Q. \)
Keep in mind (continued)

1. Determine precisely the hypothesis (if any) and the antecedent and consequent.
2. Replace (if necessary) the antecedent with a more usable equivalent.
3. Replace (if necessary) the consequent by something equivalent & more readily shown.
4. Beginning with the assumption of the antecedent, develop a chain of statements that leads to the consequent. Each statement in the chain must be deducible from its predecessors or other known results.

→ A proof of a statement symbolized by \( P \Rightarrow (Q \lor R) \) would probably have two parts:
\[
[(P \lor Q) \Rightarrow R] \iff [(P \Rightarrow R) \land (Q \Rightarrow R)]
\]

Examples

→ (Pg 29) If a proof contains the line "The product of a real numbers a and b is zero," we could assert that "Either \( a = 0 \) or \( b = 0 \)." In this example, the equivalence of the two statements comes from our knowledge of the real numbers that \((ab = 0) \iff (a = 0 \text{ or } b = 0)\).
Examples (continued)

→(pg.30)- You are at a crime scene and have established the facts:

1) If the crime did not take place in the billiard room, the Colonel Mustard is guilty.
2) The lead pipe is not the weapon.
3) Either Colonel Mustard is not guilty or the weapon used was the lead pipe.

Contracting a proof from these facts using Modus Ponen

Statement (1) \( \neg B \rightarrow M \)

(2) \( \neg L \)

(1), (2), (3) imply that \( [\neg B \rightarrow M] \wedge \neg L \wedge (\neg M \lor L) \) is a tautology.

Therefore the crime took place in the billiard room.

→(pg.36) Proof by exhaustion.

Let \( x \) be a real number. Prove that \( -1x \leq x \leq 1x \).

Proof: Since the absolute value of \( x \) is defined by cases \( (1x) = x \) if \( x \geq 0 \), \( 1x = -x \) if \( x < 0 \).

Case 1: Suppose \( x \geq 0 \). Then \( 1x = x \). Since \( x \geq 0 \), we have \(-x \leq x\). Hence, \(-x \leq x \leq x\), which is \(-1x \leq x \leq 1x\).

Case 2: Suppose \( x < 0 \). Then \( 1x = -x \). Since \( x < 0 \), \( x \leq -x \).

Hence, we have \( -x \leq x \leq -x \) or \(-1x \leq x \leq -x\).

which is \(-1x \leq x \leq 1x\).

Finally, cases: \(-1x \leq x \leq 1x\).