Basic Proof Methods

1.4

- Axioms or postulates - initial set of statements that are assumed to be true.

- Undefined terms - concepts fundamental to the context of study.

- The Replacement Rule: Rename a statement involving logical connectives.

  \[ \text{Ex: } \text{"It is not the case that } x \text{ is even and prime"} \]

  \[ \neg (P \land Q) \]

  \[ \text{"x is not even or x is not prime"} \]

  \[ \neg P \lor \neg Q \]

- Tautology rule: At any time state a sentence whose symbolic translation is a tautology.

- Modus ponens - when \( P \) and \( P \rightarrow Q \) are both true, we can conclude \( Q \) must be true.

- Direct Proof: \( P \rightarrow Q \), antecedent \( P \) to the consequent \( Q \). \( P \rightarrow Q \) is false only when \( P \) is true and \( Q \) is false.
1. Determine precisely the hypothesis and the antecedent and consequent.
2. Replace the antecedent with a more readable equivalent.
3. Replace the consequent by something equivalent and more readable shown.
4. Beginning with the assumption of the antecedent, develop a chain of statements that lead to the consequent.

Proof of exhaustion; examination of every possible case.

**Example:** Let \( x \) be a real number. Prove that
\[-|x| \leq x \leq |x|\]

**Proof:**

**Case 1:** Suppose \( x \geq 0 \). Then \( |x| = x \), since \( x \geq 0 \), we have \(-x \leq x\). Hence,
\[-x \leq x \leq x\]
which is
\[-|x| \leq x \leq |x|\]
in this case.

**Case 2:** Suppose \( x < 0 \). Then \( |x| = -x \).
Since \( x < 0 \), \( x \leq -x \). Hence, we have
\[x \leq x \leq -x\]
or \((-x) \leq x < -x\) which is
\[-|x| \leq x \leq |x|\].