Informal Summary 1.4

- **theorem** - a statement that describes a pattern or relationship among quantities or structures
- **proof** - a justification of the truth of a theorem
- **axiom (postulate)** - initial set of statements that are assumed true
- **undefined terms** - concepts fundamental to the context of study

At any time, state an assumption, an axiom, or a previously proved result.

At any time, state a sentence equivalent to any statement earlier in proof.

Ex.: Suppose we have been able to establish the step "It is not the case that x is even and prime." Replacement Rule

We can then deduce that "x is not even or x is not prime"

\[ \neg (P \land Q) \equiv \neg P \lor \neg Q \]

At any time, state a sentence whose symbolic translation is a tautology.

- **modus ponens** - based on the tautology \([P \land (P \Rightarrow Q)] \Rightarrow Q\]

At any time, if \(P \Rightarrow Q\) and \(P\) appear in a proof, state \(Q\) is true.

- **direct proof** - a statement of the form \(P \Rightarrow Q\). Prove \(P\) to consequent \(Q\) true.

L) Assume \(P\).

Therefore, \(Q\).

Thus, \(P \Rightarrow Q\).

- Strategy for developing a direct proof of a conditional statement:
  1) Determine precisely the hypotheses (if any) and the antecedent and consequent.
  2) Replace (if necessary) the antecedent with a more usable equivalent.
  3) Replace (if necessary) the consequent by something equivalent and more readily shown.
  4) Begin with the assumption of the antecedent, develop a chain of statements that leads to the consequent. Each statement in the chain must be deducible from its predecessors or other known results.
Proof by cases:

Case 1. Assume P... Therefore R.
Case 2. Assume Q... Therefore R.