A Transition to Advanced Mathematics by Smith, et.al (7th edition)

Definitions from book:

- **Theorem:** a statement that describes a pattern or relationship among quantities or structures.
- **Proof:** a justification of the truth of a theorem.
- **Axioms (or postulates):** an initial set of statements that are assumed to be true.

Rule of a proof:

In any proof you may *at any time state an assumption, an axiom, or previously proved result.*

Replacement rule:

At any time stating a sentence equivalent to any statement earlier in the proof.

Tautology rule:

At any time stating a sentence whose symbolic translation is a tautology.

Modus Ponens:

At any time after P and P => Q in a proof, state that Q was true.

- **Strategies for Developing a Direct Proof of a Conditional Sentence**
  - Determine precisely the hypothesis (if any) and antecedent and consequent.
  - Replace (if necessary) the antecedent with a more usable equivalent.
  - Replace (if necessary) the consequent by something equivalent and more readily shown.
  - Beginning with the assumption of the antecedent, develop a chain of statements that leads to the consequent. Each statement in the chain must be deducible from its predecessors or other known results.

- **Proof of Exhaustion**- examination of every possible case in an example to prove true.

EXAMPLE: (p. 30)

You are at a crime scene and have established the following facts:

1. If the crime did not take place in the billiard room, then Colonel Mustard is guilty.
2. The lead pipe is not the weapon.
3. Either Colonel Mustard is not guilty or the weapon used was a lead pipe.

We can now construct a proof based on these facts and modus ponens, and show that the crime took place in the billiard room:

Proof:

- *Statement (1)* $\sim B \Rightarrow M$
- *Statement (2)* $\sim L$
- *Statement (3)* $\sim M \vee L$
- *Statements (1) and (2) and (3)* $(\sim B \Rightarrow M) \wedge \sim L \wedge (\sim M \vee L)$
- *Statements (1),(2) and (3) Imply the crime took place in the billiard room.* $[(\sim B \Rightarrow M) \wedge \sim L \wedge (\sim M \vee L)] \Rightarrow B$ *is a tautology.*

Therefore, the crime took place In the billiard room. B

DIRECT PROOF OF P => Q

Proof:

Assume P

Therefore, Q.

Thus, P => Q.

Example from class: **(7a)**

Let a be an integer.

Prove that 2a-1 is odd:

$(\forall a \, \varepsilon \mathbb{Z})(\, 2a-1 \text{ is odd})$

$((\forall a \, \varepsilon \mathbb{Z})(\exists k \, \varepsilon \mathbb{Z})[2a-1=2k+1]$

"Find a $\varepsilon \mathbb{Z}$"

THINKING LAND:"""

Want to find k $\varepsilon \mathbb{Z}$ so that $2a-1=2k+1$

$2a-1-1=2k$

$2a-2=2k$

$a-1=k$ """

Proof: fix a $\varepsilon \mathbb{Z}$. Let $k=a-1$. Since a $\varepsilon \mathbb{Z}$; k $\varepsilon \mathbb{Z}$

Note: $2a-1=2k+1$

- So by definition of odd integer, 2a-1 is odd.