Explorations and Activities Exercise

The Importance of the Base Step

Most of the work done in an induction proof is usually in proving the inductive step. This was certainly the case in Proposition 4.2 (pg. 175). However, the basis step is an essential part of the proof. As this Exercise illustrates, an induction proof is incomplete without the Base Step.

►. Let $P(n)$ be ⟨the open sentence in the variable $n \in \mathbb{N}$⟩

$$\sum_{j=1}^{n} j = \frac{n^2+n+1}{2}. \tag{$P(n)$}$$

1. Below the dotted line, complete the proof that $P(n)$ implies $P(n+1)$ for each $n \in \mathbb{N}$. Just delete the "for you ..." parts and replacing them with the needed algebra (add/delete rows as needed).
2. Is $P(1)$ true? Is $P(2)$ true? Using Progress Check 4.3 (see pages 177 and 511), for what $n \in \mathbb{N}$ is $P(n)$ true? Explain how this shows that the basis step is an essential part of a proof by induction.

. . . .

1. **Solution for Part 1**.

Proof. Let $P(n)$ be the open sentence in the variable $n \in \mathbb{N}$

$$\sum_{j=1}^{n} j = \frac{n^2+n+1}{2}.$$

Fix $n \in \mathbb{N}$. Let $P(n)$ be true ⟨think of as the inductive hypothesis⟩. Thus

$$\sum_{j=1}^{n} j = \frac{n^2+n+1}{2}. \tag{IH}$$

We shall show that $P(n+1)$ is true ⟨think of as the inductive conclusion⟩, i.e., we shall show that

$$\sum_{j=1}^{n+1} j = \frac{(n+1)^2+(n+1)+1}{2}. \tag{IC}$$

Using (IH) and algebra we get

$$\sum_{j=1}^{n+1} j = \left[\sum_{j=1}^{n} j\right] + (n+1)$$

and by (IH) we get

$$= \left[\frac{n^2+n+1}{2}\right] + (n+1)$$

and now by algebra ⟨remember, look at where you are going for guidance on what algebra to do next⟩

= for you ... getting a common demoninator is a good start

= for you ... now keep doing simple algebra ...

= for you ... use as many lines as you need ...

= for you ... until you get the Right Hand Side of (IC) ...

$$= \frac{(n+1)^2 + (n+1) + 1}{2}.$$

We have just show that (IC) holds.

Thus, for each $n \in \mathbb{N}$, if $P(n)$ is true then $P(n+1)$ is true. $\square$

2. **Solution for Part 2**.

DELETE this whole line and then put your answer to part 2 here.