Evaluation of Proof Exercise

Following the instructions for (linked) *Evaluation of Proofs* exercises (which also are posted on the course homework page), evaluate the below justification of the given conjecture.

►. **Conjecture A**. For all nonzero integers a and b, if $a+2b \neq 3$ and $9a+2b \neq 1$, then the equation $ax^3+2bx=3$ does not have a solution that is a natural number.
Hint. Conjecture A can be written symbolically as

$$\left(\forall\,(a,b) \in \mathbb{Z}^{\neq 0} \times \mathbb{Z}^{\neq 0}\right)\ \left[\ (a+2b \neq 3 \wedge 9a+2b \neq 1) \implies (\forall x \in \mathbb{N})\left[\ ax^3+2bx \neq 3\ \right]\ \right].$$

Using the contrapositive and DeMorgan, Conjecture A is logically equivalent to the statement

$$\left(\forall\,(a,b) \in \mathbb{Z}^{\neq 0} \times \mathbb{Z}^{\neq 0}\right)\ \left[\ (\exists x \in \mathbb{N})\left[\ ax^3+2bx = 3\ \right] \implies (a+2b = 3 \vee 9a+2b = 1)\ \right].$$

Proposed Proof. We will show that Conjecture A is true by proving Conjecture A's contrapositive, which is

> For all nonzero integers a and b, if the equation $ax^3+2bx=3$ has a solution that is a natural number, then $a+2b=3$ or $9a+2b=1$.

So we let a and b be nonzero integers and assume that the natural number n is a solution of the equation $ax^3+2bx=3$. So we have

$$an^3+2bn=3,$$

which gives

$$n\left(an^2+2b\right)=3.$$

So we can conclude that $n=3$ and $an^2+2b=1$. Since we now have the value of n, we can substitute it in the equation $an^3+2bn=3$ and obtain $27a+6b=3$. Dividing both sides of this equation by 3 shows that $9a+2b=1$.

So there is no need for us to go any further, and this concludes the proof of the contrapositive of Conjecture A. □

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DELETE this whole sentence and THEN put your answer to ALL parts down here.