►. **Theorem 1**. There are no natural numbers j and n with $n \geq 2$ and $j^2 + 1 = 2^n$.
Prove Theorem 1.

Hints: Thm. 1 can be expressed as

$$\sim (\exists j \in \mathbb{N}) \left(\exists n \in \mathbb{N}^{\geq 2}\right) \left[\ j^2 + 1 = 2^n\ \right]. \tag{WTS}$$

So a negation of Thm. 1 is ⟨since $\sim [\sim P] \equiv P$⟩

$$(\exists j \in \mathbb{N}) \left(\exists n \in \mathbb{N}^{\geq 2}\right) \left[\ j^2 + 1 = 2^n\ \right]. \tag{∼WTS}$$

For a proof by contradiction, we would assume (∼WTS) hold true and look for a contradiction. Let's say in the case that j is even, we could find a contradiction. And in the case that j is odd, we could find another contradiction. Then either case (j even or odd) leads to a contradiction. Furthermore, the 2 cases are exhaustive. Note if a number can be written as 2^n, then you know something about the parity, as well as prime factorization, of that number.

. . . .

DELETE this whole sentence and THEN put your answer to ALL parts down here.