

The symbol for the rational numbers is \mathbb{Q} while the symbol for the irrational numbers is $\mathbb{R} \setminus \mathbb{Q}$.
So you can express that x is an irrational number by $x \notin \mathbb{Q}$ or by $x \in \mathbb{R} \setminus \mathbb{Q}$.
Recall for any sets R and Q , the set R set minus Q is the set $R \setminus Q$ $\stackrel{\text{def.}}{=} \{x \in R: x \notin Q\}$.
Note the difference in direction in the backslash for set minus ($R \setminus Q$) and quotient of numbers ($1/2 = 0.5$).

►. **Proposition 3.19.** Symbolically written. (TS book §3.3 page 123)

$$(\forall (x, y) \in \mathbb{R}^2) [(x \in \mathbb{Q} \wedge x \neq 0 \wedge y \notin \mathbb{Q}) \implies xy \notin \mathbb{Q}] \tag{1}$$

1. We have discussed closure properties of number systems (e.g., \mathbb{R} , \mathbb{Q} , $\mathbb{R} \setminus \mathbb{Q}$) under certain operations (e.g., addition, multiplication, division by a nonzero number). Proposition 3.19 gives that the irrational numbers are closed under a certain operation. What is this operation?

Hint: Complete sentences. “The irrational numbers are closed under the operation of ??????”.

2. Using that $\sim [P \implies Q] \equiv [P \wedge (\sim Q)]$ (idea: negation of a promise is a lie), symbolically write a negation (denial) if the statement in (1). Your answer can not contain the negation symbol \sim (nor any variant of the \sim symbol). Your answer can contain the symbols \in and \notin .

hint. You may use Proposition 3.19 in the ER’s from §3.3 and beyond.

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DELETE this whole sentence and THEN put your answer to ALL parts down here.