►. Let y_1, y_2, y_3, y_4 be real numbers. The **mean**, $\overline{y}$, of these four numbers is defined to be the sum of the four numbers divided by 4. That is,

$$\overline{y} = \frac{y_1 + y_2 + y_3 + y_4}{4}.$$

Prove that there exists a y_i with $1 \leq i \leq 4$ such that $\overline{y} \leq y_i$.

Hint. Symbolically written, this problem says

$$\left(\forall (y_1, y_2, y_3, y_4) \in \mathbb{R}^4\right) \left(\exists i \in \{1, 2, 3, 4\}\right) \left[\frac{y_1 + y_2 + y_3 + y_4}{4} \leq y_i \right].$$

Vaguely speaking, this problem says: (the average of 4 real numbers) $\leq$ (the largest of those 4 real nubmers). One way to show this is to let $y_{\max}$ be the largest of y_1, y_2, y_3, y_4. The notation $y_{\max}$ should greatly help. Think about what kind of inequalitites can you get between $y_{\max}$ and the y_i's.

DELETE this whole sentence and THEN put your answer to ALL parts down here.