

Recall (a.k.a., hint) you may use either below (equivalent) formulation as the definition of rational number.

**Def. 1.** A real number  $x$  is rational provided

$$(\exists a \in \mathbb{Z}) (\exists b \in \mathbb{Z}) \left[ x = \frac{a}{b} \wedge b \neq 0 \right].$$

**Def. 2.** A real number  $x$  is rational provided

$$(\exists a \in \mathbb{Z}) (\exists b \in \mathbb{N}) \left[ x = \frac{a}{b} \right]$$

Def. 1 appeared on p11. Def. 2 is often easier to work with.

►. **Theorem 1.** If  $p, q \in \mathbb{Q}$  with  $p < q$ , then there exists  $x \in \mathbb{Q}$  with  $p < x < q$ .

1. Symbolically write Theorem 1. Hint. Your answer should be of the form

$$\left( \forall (p, q) \in \mathbb{Q}^2 \right) [p < q \implies (\exists x \in \mathbb{Q}) [? \text{what goes here?} ]].$$

2. Prove Theorem 1. You may use the closure properties of  $\mathbb{Q}$  that we covered in Chapter 1.

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DELETE this whole sentence and THEN put your answer to ALL parts down here.