Closure Explorations

Explorations and Activities

In this exercise, you may use (without proof) the following prior results from the (linked) Ch. 1 class handout.

Previously Shown Results

**Lemma SEE**. If $x$ is an even integer and $y$ is an even integer, then $x + y$ is an even integer.
**Lemma SEO**. If $x$ is an even integer and $y$ is an odd integer, then $x + y$ is an odd integer.
**Lemma SOO**. If $x$ is an odd integer and $y$ is an odd integer, then $x + y$ is an even integer.
**Lemma PEA**. If $x$ is an even integer and $y$ is an integer, then $x \cdot y$ is an even integer.
**Lemma POO**. If $x$ is an odd integer and $y$ is an odd integer, then $x \cdot y$ is an odd integer.

Division Algorithm

**Division Algorithm**. For all $n \in \mathbb{N}$ and $a \in \mathbb{Z}$, there exist unique integers $q$ and $r$ so that

$$a \;=\; nq \;+\; r \qquad \text{and} \qquad 0 \le r < n\ .$$

In §1.1, we studied closure properties of standard number systems (e.g. $\mathbb{Z}$ and $\mathbb{Q}$, see p. 11–12 and 32).
We can extend the closure idea to other subsets $S$ of real numbers. We say that
- A subset $S$ of real numbers is closed under addition provided that if $x$ and $y$ are in the set $S$, then $x + y \in S$.
- A subset $S$ of real numbers is closed under multiplication provided that if $x$ and $y$ are in the set $S$, then $x \cdot y \in S$.
- A subset $S$ of real numbers is closed under subtraction provided that if $x$ and $y$ are in the set $S$, then $x - y \in S$.

►. Consider the below subsets of of the real numbers defined by: ⟨note $T$ is not in set builder notation⟩

$O$ is the set of all odd integers

$E$ is the set of all even integers

$$T \;=\; \{3n + 2 \in \mathbb{Z} \colon n \in \mathbb{Z}\} \overset{\text{i.e.}}{=} \{\ldots, -7. -4, -1, 2, 5, 8, \ldots\}.$$

Below the dotted line, answer each question YES or NO. Then justify your Yes/No answer by either

- (for yes) explaining which and why prior class result ⟨from above box⟩ say yes (no formal proof needed)
- (for no) providing an counterexample showing the answer is no.

O.1. Is $O$ closed under addition?
O.2. Is $O$ closed under multiplication?
O.3. Is $O$ closed under substraction?
E.1. Is $E$ closed under addition?
E.2. Is $E$ closed under multiplication?
E.3. Is $E$ closed under substraction?
T.1. Is $T$ closed under addition?
T.2. Is $T$ closed under multiplication?
T.3. Is $T$ closed under substraction?

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DELETE this whole sentence and THEN put your answer to ALL parts down here.