Evaluation of Proof Exercise

Following the instructions for (linked) *Evaluation of Proofs* exercises (which also are posted on the course homework page), evaluate the below justification of the given conjecture.

►. **Conjecture B**. If m is an integer, then 5 divides $(m^5 - m)$.

Proposed Proof. We shall show that Conjecture B is true. Let $m \in \mathbb{Z}$. We will prove that 5 divides $(m^5 - m)$ by proving that $(m^5 - m) \equiv 0 \pmod 5$. We will use cases.

For Case 1, let

$$m \equiv 0 \pmod 5.$$

Modulo arithmetic gives $(m^5 - m) \equiv (0^5 - 0) \pmod 5$. Thus $(m^5 - m) \equiv 0 \pmod 5$ in Case 1.

For Case 2, let

$$m \equiv 1 \pmod 5.$$

Modulo arithmetic gives $(m^5 - m) \equiv (1^5 - 1) \pmod 5$. Thus $(m^5 - m) \equiv 0 \pmod 5$ in Case 2.

For Case 3, let

$$m \equiv 2 \pmod 5.$$

Then, using modulo arithmetic, we get $m^5 - m \equiv 2^5 - 2 \pmod 5$. Note $2^5 - 2 = 32 - 2$ and so

$$m^5 - m \equiv 30 \pmod 5. \tag{50}$$

Since $30 = (5)(6) + 0$

$$30 \equiv 0 \pmod 5. \tag{51}$$

Since congruence is transitive, (50), and (51) gives $m^5 - m \equiv 0 \pmod 5$. This finishes Case 3.

We have just shown Conjecture B is true by using proof by cases. □

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