Evaluation of Proof Exercise

Following the instructions for (linked) *Evaluation of Proofs* exercises (which also are posted on the course homework page), evaluate the below justification of the given conjecture.

►. **Conjecture 22A**. For all integers a and b, if $(a+2b) \equiv 0 \pmod 3$, then $(2a+b) \equiv 0 \pmod 3$.

Proposed Proof. We shall show that Conjecture 22A is true. We assume $a, b \in \mathbb{Z}$ and $(a+2b) \equiv 0 \pmod 3$. We shall show that $(2a+b) \equiv 0 \pmod 3$.

Since $(a+2b) \equiv 0 \pmod 3$, we know 3 divides $a+2b$. Hence, there exists $m \in \mathbb{Z}$ such that

$$a + 2b = 3m. \tag{16}$$

Hence

$$a = 3m - 2b. \tag{17}$$

For $(2a+b) \equiv 0 \pmod 3$, there exists an integer x such that

$$2a + b = 3x. \tag{18}$$

Hence by equations (17) and (18) and then algebra

$$\begin{aligned} 2(3m-2b) + b &= 3x \\ 6m - 3b &= 3x \\ 3(2m-b) &= 3x \\ 2m - b &= x. \end{aligned}$$

Since $(2m-b) \in \mathbb{Z}$ this proves that 3 divides $(2a+b)$ and hence, $(2a+b) \equiv 0 \pmod 3$. □

hint. Symbolically written Conjecture 22A is

$$\left(\forall (a,b) \in \mathbb{Z}^2\right) \left[\overbrace{(a+2b) \equiv 0 \pmod 3}^{\text{hypothesis}} \implies \overbrace{(2a+b) \equiv 0 \pmod 3}^{\text{conclusion}} \right].$$

hint. (16) $\iff a + 2b = 3m \iff 2a + 4b = 6m \iff 2a + b + 3b = 6m$.

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