

Evaluation of Proof Exercise

Following the instructions for [\(linked\)](#) *Evaluation of Proofs* exercises (which also are posted on the course homework page), evaluate the below justification of the given conjecture.

- . **Conjecture A.** For all nonzero integers a and b , if $a + 2b \neq 3$ and $9a + 2b \neq 1$, then the equation $ax^3 + 2bx = 3$ does not have a solution that is a natural number.

Hint. Conjecture A can be written symbolically as

$$(\forall (a, b) \in \mathbb{Z}^{\neq 0} \times \mathbb{Z}^{\neq 0}) \left[(a + 2b \neq 3 \wedge 9a + 2b \neq 1) \implies (\forall x \in \mathbb{N}) \left[ax^3 + 2bx \neq 3 \right] \right].$$

Using the contrapositive and then DeMorgan, Conjecture A is logically equivalent to the statement

$$(\forall (a, b) \in \mathbb{Z}^{\neq 0} \times \mathbb{Z}^{\neq 0}) \left[(\exists x \in \mathbb{N}) \left[ax^3 + 2bx = 3 \right] \implies (a + 2b = 3 \vee 9a + 2b = 1) \right].$$

Proposed Proof. We will show that Conjecture A is true by proving Conjecture A's contrapositive, which is

For all nonzero integers a and b , if the equation $ax^3 + 2bx = 3$ has a solution that is a natural number, then $a + 2b = 3$ or $9a + 2b = 1$.

So we let a and b be nonzero integers and assume that the natural number n is a solution of the equation $ax^3 + 2bx = 3$. So we have

$$an^3 + 2bn = 3,$$

which gives

$$n(an^2 + 2b) = 3.$$

So we can conclude that $n = 3$ and $an^2 + 2b = 1$. Since we now have the value of n , we can substitute it in the equation $an^3 + 2bn = 3$ and obtain $27a + 6b = 3$. Dividing both sides of this equation by 3 shows that $9a + 2b = 1$.

So there is no need for us to go any further, and this concludes the proof of the contrapositive of Conjecture A. □

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