►. **Theorem 8**. There are no natural numbers j and n with $n \geq 2$ and $j^2 + 1 = 2^n$.

1. Prove Theorem 8.

hint. Do we know something about the parity (even/odd-ness) of a number of the form 2^s for some $s \in \mathbb{N}$?

hint. Thm. 8 can be expressed symbolically as

$$\sim (\exists j \in \mathbb{N}) \left(\exists n \in \mathbb{N}^{\geq 2}\right) \left[\ j^2 + 1 = 2^n\ \right]. \tag{WTS}$$

So a negation of Thm. 8 is $\langle \text{since} \sim [\sim P] \equiv P \rangle$

$$(\exists j \in \mathbb{N}) \left(\exists n \in \mathbb{N}^{\geq 2}\right) \left[\ j^2 + 1 = 2^n\ \right]$$

or, since there are two ∃'s in a row they can be interchanged, is

$$\left(\exists n \in \mathbb{N}^{\geq 2}\right) (\exists j \in \mathbb{N}) \left[\ j^2 + 1 = 2^n\ \right] \tag{$\sim$WTS}$$

For a proof by contradiction, we would assume (∼WTS) holds true and look for a contradiction. Let's say in the case that j is even we could find a contradiction while in the case that j is odd we could find another contradiction. Then either case (j even or odd) leads to a contradiction. Furthermore, theses two cases for j are exhaustive (i.e. cover all possible cases for j).

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