

One of the *proof methods* we learned in §3.2 was *proof using logically equivalent statements*. Often we have used this method to give a proof of a conditional statement $P \Rightarrow Q$ by proving the logically equivalent contrapositive $\sim Q \Rightarrow \sim P$. This ER uses different logical equivalence from the (linked) handout [§2.2 Logically Equivalent Statements](#).

►. **Theorem 5.** For all integers a, b , and d with $d \neq 0$, if d divides a or d divides b , then d divides the product ab .

1. Symbolically write Theorem 5. Hint: below is a start.

$$(\forall (a, b, d) \in \mathbb{Z} \times \mathbb{Z} \times \mathbb{Z}^{\neq 0}) [(d \mid a \vee d \mid b) \implies \underline{\hspace{2cm} ? \hspace{2cm}}] \quad (1)$$

2. Symbolically write an equivalent statement to the statement in (1) by using one of the *Important Logical Equivalencies* from the (linked) handout [§2.2 Logically Equivalent Statements](#).

Hint: don't forget quantifiers, so answer should take the below form.

$$(\forall (a, b, d) \in \mathbb{Z} \times \mathbb{Z} \times \mathbb{Z}^{\neq 0}) [\underline{\hspace{2cm} ? \hspace{2cm}}] \quad (2)$$

3. Prove Theorem 5.

Hint: to prove Thm. 5, we can prove either (1) or the equivalent (2). Which way looks easier to prove?

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